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Neutrosophic Economic Order Quantity: Backlogged Shortages and Quality Issues

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***Abstract* – This paper investigates an economic order quantity with imperfect quality items that are backlogged in the neutrosophic sense. Defuzzification is done by implementing the signed-distance approach. The objective is to determine the optimal inventory level and optimal backorder quantity that reduces the yearly total cost of the neutrosophic type. Numerical examples are produced to justify the output of the suggested models.**

Keywords—Neutrosophic Economic Order Quantity, Neutrosophic Number, Minimal Total Cost, Shortfall Quantity.

I. INTRODUCTION

The product received (or) produced in classic economic order quantity models is implicitly anticipated to be of perfect quality. However, this isn't always the case. The lot sizes are determined by the manufacturing process, natural hazards, detriment in transport, or a variety of other factors. Several authors created production/inventory models using the aforementioned scenarios to better represent real-world situations, and they also looked at how lot size rules are affected by faulty quality. The previously stated implicit assumption is legitimate, especially in cases when substandard products are manufactured. Typically, they are identified during the screening procedure and promptly removed from inventory. In the event of a shortfall in supply, it is assumed that all consumers are willing to wait. The economic order quantity (EOQ) model [1] was presented for inadequate-quality commodities. An economic production quantity model (EPQ) with commodities of inadequate quality [2] was designed based on the production criterion. The key reasons to adapt EOQ for imperfect items are as follows: (a) Quality control and inspection cost, (b) Rework and disposal cost, (c) Inventory holding cost, (d) Order quantity adjustments, (e) Supplier relationship and lead time, (f) Customer satisfaction and service level. Incorporating a fuzzy model into the Economic Order Quantity (EOQ) analysis for an inventory system can help address the uncertainties and vagueness present in real-world scenarios. Traditional EOQ models often rely on precise input parameters such as demand rate, holding costs, and ordering costs. However, these parameters can be uncertain or imprecise due to factors such as fluctuating demand, variable lead times, and imprecise cost estimations. Fuzzy logic provides a framework to handle such uncertainties. An economic production lot size model [3] was

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demonstrated in the context of an imperfect production system. An economic production quantity model for items with poor quality [4] has been furnished and a practical solution to the problem was attained. Later, an EOQ model that includes faulty products and shortfalls [5] was structured. With the above models as a reference, an EOQ model for commodities with poor quality and inspection faults [6] was developed. The usage of fuzzy set theory in an EOQ model for commodities with poor quality and scarce backorder [7] emerged. A precise examination of the fuzzy-based EOQ with defective products, shortfalls, and faults on inspection [8] has been analyzed. The EOQ model was further extended to deal with an intuitionistic fuzzy environment with imperfect quality products, where shortages are backlogged [9]. A new set theory named, "neutrosophic set" [10] was defined and treated as the modulated and updated form of the preceding fuzzy models. A suggested inventory model that does not include shortages [11] has been executed. The initiation of the EOQ model in a neutrosophic environment with price breaks [12] was put forth. Also, the formulation of a neutrosophic inventory problem that deals with backorder using the triangular neutrosophic number [13] was dealt with. A single-valued neutrosophic inventory model is processed with a neutrosophic random variable [14]. The construction of a neutrosophic inventory model has been done to treat deficient [15] and decaying items [16] under immediate return and price-dependent demand, respectively. A neutrosophic logic was taken and applied to an inventory model dealing with credit policy for perishable products [17]. The structure of an inventory control problem using the extended uncertainty principle [18] has been recorded. The incorporation of a neutrosophic inventory system that includes a neutrosophic cost pattern with deterioration [19] and a discount on defective items [20] using a particle swarm algorithm is listed. The composition of a neutrosophic inventory that involves reworking faulty products with three carbon policies [21] has been incorporated.

This paper considers an economic order quantity with imperfect quality items that are backlogged, where holding cost, shortfall cost, transportation cost, and inspection cost are taken as triangular neutrosophic numbers. Incorporating imperfect items into the EOQ model involves modifying the standard EOQ formula to reflect the reality of quality imperfections. To minimize the annual neutrosophic total cost per unit time, the method of signed distance is used to evaluate the estimate of the overall cost per unit of time in the neutrosophic sense, and then the related optimal order quantity and optimal backorder quantity are derived. To study the behavior of our proposed models, numerical examples are carried out.

II. NOTATIONS

C_h^N - Carrying cost of neutrosophic type per unit quantity per unit of time

C_s^N - Shortfall cost of neutrosophic type per Unit quantity per unit of time

C_i^N - Inspection cost of neutrosophic type

C_t^N - Transportation cost of neutrosophic type

D^N - Total demand for the neutrosophic type

$(TC)^N$ - Total cost of neutrosophic type

Q^N - Order quantity of neutrosophic type

T ^{N} - Cycle length of neutrosophic type

III. ASSUMPTIONS

- At the beginning of each neutrosophic inventory cycle, only one order is produced.
- In this context, Q^N represents the neutrosophic lot size for each cycle, s_1^N denotes the beginning inventory level of neutrosophic after completing the back-logged amount from the previous cycle, and $Q^N - s_1^N$ signifies the maximum shortfall level.

IV. MODEL DESCRIPTION

In this model, holding cost, shortage cost, inspection cost, and transportation cost are represented as equations (1), (2), (3), and (4) respectively, in triangular neutrosophic numbers.

Let

$$C_h^N = (C_{h_1}^N, C_{h_2}^N, C_{h_3}^N)(C_{h_1}{}'^N, C_{h_2}^N, C_{h_3}{}'^N)(C_{h_1}{}''^N, C_{h_2}^N, C_{h_3}{}''^N) \quad (1)$$

$$C_s^N = (C_{s_1}^N, C_{s_2}^N, C_{s_3}^N)(C_{s_1}{}'^N, C_{s_2}^N, C_{s_3}{}'^N)(C_{s_1}{}''^N, C_{s_2}^N, C_{s_3}{}''^N) \quad (2)$$

$$C_i^N = (C_{i_1}^N, C_{i_2}^N, C_{i_3}^N)(C_{i_1}{}'^N, C_{i_2}^N, C_{i_3}{}'^N)(C_{i_1}{}''^N, C_{i_2}^N, C_{i_3}{}''^N) \quad (3)$$

$$C_t^N = (C_{t_1}^N, C_{t_2}^N, C_{t_3}^N)(C_{t_1}{}'^N, C_{t_2}^N, C_{t_3}{}'^N)(C_{t_1}{}''^N, C_{t_2}^N, C_{t_3}{}''^N) \quad (4)$$

The total cost of the neutrosophic type is illustrated by equation (5) and further simplified by equations (6) and (7).

$$(TC)^N = \frac{1}{T}\left[\frac{C_h^N s_1^{2N}}{2D^N} + \frac{1}{2D^N} C_s^N (Q^N - s_1^N)^2 + \frac{D^N}{Q^N} C_i^N + \frac{D^N}{Q^N} C_t^N\right] \quad (5)$$

$$=\left(C^N_{h_1},C^N_{h_2},C^N_{h_3}\right)\left(C_{h_1}{''}^N,C^N_{h_2},C_{h_3}{''}^N\right)\frac{s_1^{2^N}}{2D^N}+\frac{(Q^N-s_1^N)^2}{2D^N}\left(C^N_{s_1},C^N_{s_2},C^N_{s_3}\right)\left(C_{s_1}{''}^N,C^N_{s_2},C_{s_3}{''}^N\right)+\frac{D^N}{Q^N}\left(C^N_{i_1},C^N_{i_2},C^N_{i_3}\right)\left(C_{i_1}{''}^N,C^N_{i_2},C_{i_3}{''}^N\right)+\frac{D^N}{Q^N}\left(C^N_{t_1},C^N_{t_2},C^N_{t_3}\right)\left(C_{t_1}{''}^N,C^N_{t_2},C_{t_3}{''}^N\right) \quad (6)$$

$$=\Bigg(C^N_{h_1}\frac{s_1^{2^N}}{2D^N}+\frac{(Q^N-s_1^N)^2}{2D^N}C^N_{s_1}+\frac{D^N}{Q^N}C^N_{i_1}+\frac{D^N}{Q^N}C^N_{t_1},C^N_{h_2}\frac{s_1^{2^N}}{2D^N}+\frac{(Q^N-s_1^N)^2}{2D^N}C^N_{s_2}+\frac{D^N}{Q^N}C^N_{i_2}+\frac{D^N}{Q^N}C^N_{t_2},C^N_{h_3}\frac{s_1^{2^N}}{2D^N}+\frac{(Q^N-s_1^N)^2}{2D^N}C^N_{s_3}+\frac{D^N}{Q^N}C^N_{i_3}+\frac{D^N}{Q^N}C^N_{t_3}\Bigg)\Bigg(C_{h_1}{''}^N\frac{s_1^{2^N}}{2D^N}+\frac{(Q^N-s_1^N)^2}{2D^N}C_{s_1}{''}^N+\frac{D^N}{Q^N}C_{i_1}{''}^N+\frac{D^N}{Q^N}C_{t_1''^N},C^N_{h_2}\frac{s_1^{2^N}}{2D^N}+\frac{(Q^N-s_1^N)^2}{2D^N}C^N_{s_2}+\frac{D^N}{Q^N}C_{i_2}{''}^N+\frac{D^N}{Q^N}C_{t_2}{''}^N,C_{h_3}{''}^N\frac{s_1^{2^N}}{2D^N}+\frac{(Q^N-s_1^N)^2}{2D^N}C_{s_3}{''}^N+\frac{D^N}{Q^N}C_{i_3}{''}^N+\frac{D^N}{Q^N}C_{t_3}{''}^N\Bigg) \quad (7)$$

The total cost of neutrosophic defuzzification using the signed distance approach is expressed by the equation (8):

$$F(q)^N=\frac{1}{12}\Big[\Big(C^N_{h_1}\frac{s_1^{2^N}}{2D^N}+\frac{(Q^N-s_1^N)^2}{2D^N}C^N_{s_1}\Big)+2\Big(C^N_{h_2}\frac{s_1^{2^N}}{2D^N}+\frac{(Q^N-s_1^N)^2}{2D^N}C^N_{s_2}\Big)+\Big(C^N_{h_3}\frac{s_1^{2^N}}{2D^N}+\frac{(Q^N-s_1^N)^2}{2D^N}C^N_{s_3}\Big)+\Big(C_{h_1}{''}^N\frac{s_1^{2^N}}{2D^N}+\frac{(Q^N-s_1^N)^2}{2D^N}C_{s_1}{''}^N\Big)+2\Big(C^N_{h_2}\frac{s_1^{2^N}}{2D^N}+\frac{(Q^N-s_1^N)^2}{2D^N}C^N_{s_2}\Big)+\Big(C_{h_3}{''}^N\frac{s_1^{2^N}}{2D^N}+\frac{(Q^N-s_1^N)^2}{2D^N}C_{s_3}{''}^N\Big)\Big] \quad (8)$$

To find the minimum of $D(F(q)^N)$, take the derivative $D(F(q)^N)$ using equation (8) and equating the derivative $D(F(q)^N)$ to zero, then we have equation (9) as follows:

$$\frac{1}{12}\Big\{\frac{s_1}{D}\big[(C_{h_1}+C_{s_1})+4(C_{h_2}+C_{s_2})+(C_{h_3}+C_{s_3})+(C_{h_1}''+C_{s_1}'')+(C_{h_3}''+C_{s_3}'')\big]-\frac{Q}{D}\big[C_{s_1}+4C_{s_2}+C_{s_3}+C_{s_1}''+C_{s_3}''\big]\Big\}=0 \quad (9)$$

On simplifying (9), we get equation (10),

$$s_1^N=\frac{C_{s_1}+4C_{s_2}+C_{s_3}+C_{s_1}''+C_{s_3}''}{(C_{h_1}+C_{s_1})+4(C_{h_2}+C_{s_2})+(C_{h_3}+C_{s_3})+(C_{h_1}''+C_{s_1}'')+(C_{h_3}''+C_{s_3}'')}Q \quad (10)$$

Equation (10) is modified to equation (11) as follows:

$$s_1^N=\frac{C_{s_1}+4C_{s_2}+C_{s_3}+C_{s_1}''+C_{s_3}''}{(C_{h_1}+C_{s_1})+4(C_{h_2}+C_{s_2})+(C_{h_3}+C_{s_3})+(C_{h_1}''+C_{s_1}'')+(C_{h_3}''+C_{s_3}'')}DT \quad (11)$$

V. NUMERICAL EXAMPLE

A company's demand for a product is 50 units each day. If there is a shortage, the holding fee per day is Rs. 20, and if it is delayed to meet the scheduled shipping date, the shortage cost is Rs. 60. The inspection and transportation costs are Rs. 6 and Rs. 80 respectively. Determine the optimal annual inventory level in a neutrosophic environment.

Solution:

D^N =50,
C_s^N =(10,60,380)(45,60,120)(30,60,240),
C_h^N =(5,20,250)(15,20,100)(12,20,180)
C_i^N=(1,6,75)(5,6,20)(4,6,50),
C_t^N =(40,80,360)(70,80,160)(60,80,240)

The results are then summarized as follows:

TABLE 1. Analysis of crisp & fuzzy models.

Components	Models			
	Crisp model	Fuzzy model	Intuitionistic fuzzy model	Neutrosophic model
Demand	50	50	50	50
Shortage cost	60	(10,60,380)	(10,60,380) (45,60,120)	(10,60,380) (45,60,120) (30,60,240)
Holding cost	20	(5,20,250)	(5,20,250) (15,20,100)	(5,20,250) (15,20,100) (12,20,180)
Inspection cost	6	(1,6,75)	(1,6,75) (5,6,20)	(1,6,75) (5,6,20) (4,6,50)
Transport-ation cost	80	(40,80,360)	(40,80,360) (70,80,160)	(40,80,360) (70,80,160) (60,80,240)
Optimal level of inventory	18.25	18.24	18.35	18.78
Total cost	125134.16	764386.99	4221333.81	2647431.7
Back-ordering quantity	13.687	11.55	13.81	11.35

A. Eroglu and G. Ozdemir's work [5] is used to determine the optimal inventory level, shortfall amount, and minimal total cost for the crisp, fuzzy, and intuitionistic fuzzy model. Moreover, these factors are evaluated concerning the optimal inventory level for neutrosophic circumstances, the extent of neutrosophic shortfall, and the minimal overall cost associated with neutrosophic conditions.

VI. SENSITIVITY ANALYSIS

In this section, the analysis of the optimal level of inventory, shortfall quantity, and minimal total cost for the crisp, fuzzy, and extension models, for Table 1 is shown graphically in Fig.1.

Observations:

- According to the analysis and output values derived from the gathered data, it can be seen that the optimal amount of inventory observed in the neutrosophic environment is more similar to that observed in the crisp, fuzzy, and intuitionistic environments.
- The decrease in optimum total cost in the neutrosophic model as the intuitionistic optimal total cost enhances.
- The decrease in Shortfall amount recorded in the neutrosophic model, illustrates that the shortfall quantity in the intuitionistic fuzzy model improves.

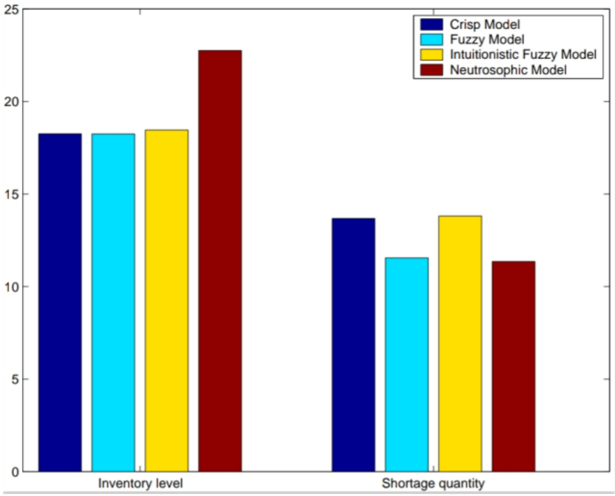

FIGURE 1. Neutrosophic inventory level and neutrosophic backorder quantity.

VII. CONCLUSION

This paper demonstrates an economic order quantity with imperfect quality items that are backlogged in the neutrosophic sense by employing triangular neutrosophic numbers. This investigation will use a current model to determine the most effective inventory level and backorder quantity to minimize the annual total cost of neutrosophic techniques. When ordered lots contain some defective items, the standard EOQ methodology is ineffective. As a result, new models are needed to provide more realistic approaches to particular situations. When each ordered lot has some defective products and shortages backorder in this article, a neutrosophic EOQ model is created. Due to different system uncertainties, stockouts are unavoidable in many real-world situations. As a result, inventory shortages could be regarded as a natural occurrence. Our forthcoming interest in neutrosophic inventory research is in the area of inspection mistakes and partial backlogs.

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