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Complement Properties of Pythagorean Co-Neutrosophic Graphs

Govindan Vetrivel, Murugappan Mullai*, Grienggrai Rajchakit*, R Surya and Saravanan Subraja

***Abstract* – The origination of graphs with neutrosophic type where membership of indeterminacy expels the vague results, by increasing the accuracy is used to extend application through the graphical environment. Since it is an extension of the intuitionistic type, there comes an immediate need to extend its findings and application to the neutrosophic type. Reversing the conditions of neutrosophic graphs by introducing the anti-behavior properties will produce an adequate number of new results and data, breaking the backlog in approaching decision-making problems and other real-world applications. This research aims to recognize the complementation concept in the Pythagorean co-neutrosophic graph, which has not been dealt with yet. The co-neutrosophic graph is the reversal concept of neutrosophic graphs, where the vertex and edge membership conditions are reversed, but the total sum of these memberships remains the same. Here, the discussion about complementation, co-complementation, and its properties are carried out on a Pythagorean co-neutrosophic Graph. As a result, an application with improved accuracy result will be obtained as an outcome.**

Keywords—Neutrosophic Graph, Pythagorean Co-Neutrosophic Graph, Strong Pythagorean Co-Neutrosophic Graph, Regular Pythagorean Co-Neutrosophic Graph, Complement of Pythagorean Co-Neutrosophic Graph.

I. INTRODUCTION

The uncertainty and ambiguous results in crisp graph theory concepts have been rectified after the innovative fuzzy work [1] done on graph theory. The fuzzy subset theory [2] emerges, which is useful for creating fuzzy graphs. The demonstration of the models on fuzzy graphs [3] came into existence. The work on fuzzy hypergraphs [4] gives a clear vision of a fuzzy environment. A strong base was laid for fuzzy graphs by extending the crisp theories and concept extension to the fuzzy field. An illustration with the initial work on the anti-fuzzy graph structure [5], and its characteristics were learnt. Later, a foundation was deeply laid with a new and improvised set theory in the name of an "intuitionistic fuzzy set" [6], where non-existence membership is inserted first and then extended to a graphical outline. An outline of a new sketch was generated on an intuitionistic fuzzy graph (IFG) idea [7] and some operations were carried out [8]. New results and techniques based on the intuitionistic type of vertices and edges were formulated. A proposal regarding some structural components and complementary features of IFG [9], [10] was developed. Numerous characteristics of Co-IFG [11] were discussed in detail. When applied to real-life problems, this theory has some inaccurate

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results. To enhance the accuracy during application to real life, an expansion of the IFS to a neutrosophic set [12-13] is done and its application can be seen in [14]. Here, is a discovery of a membership entitled “indeterminacy”, which concerns uncertain situations and environments in real-life applications is considered. A joint work to develop a neutrosophic graph [15] was completed successfully and its type representation was given. The labeling concept on neutrosophic graphs [16] was newly started. Inclusion of the anti-behavior feature in neutrosophic graphs [17] is done and its properties are utilized. An improvised membership grade arose by introducing Pythagorean membership grade in decision making [18-19], and also, an extension to Pythagorean fuzzy set theory [20] happened. With the definition of the Pythagorean fuzzy set (PFS) [21-24], the introduction of the Pythagorean fuzzy graph [25], and its properties such as notions, energy, etc. came into existence. Later, the Pythagorean neutrosophic set arose and paved the path to construct Pythagorean neutrosophic fuzzy graphs (PNG) [26]. Also, a discussion regarding the labeling conditions of PNG [27] is carried out. Some product operations and regularity in PNG [28], [29] are illustrated and the planar graph is considered in the PNG environment [30]. As an extension, we newly demonstrate the Co-behavior property in PNG i.e., Pythagorean Co-Neutrosophic Graphs.

In this paper, Pythagorean Co-Neutrosophic Graphs (PCNG) are considered, and the complement & anti-complement properties of PCNG have been executed with some basic graph definitions and theorems. This is a novel approach since complement and anti-complement properties were not applied to PCNG before. A graphical representation with additional memberships and reversing properties yields a greater significance in the area of PCNG application. The sectional components of this paper discuss the following: A detailed origin and development from fuzzy to neutrosophic graphs have been illustrated in Section I. Section II comprises definitions and examples, which are useful to elaborate on the proposed topic. Section III covers the implementation of complement properties in strong PCNG. A co-complement behavior on PCNG was briefly dealt with in Section IV. An application that relates the concept to real life has been given in Section V. Section VI encloses the final results and future ideas to extend the work.

II. PRELIMINARIES

Definition 2.1

A Pythagorean Co-Neutrosophic graph (PCNG) is described of the form Gr = ⟨A, B⟩, where

(i) A = $\{a_1, a_2, \ldots, a_n\}$ such that Ex_1 : A → [0, 1], Un_1 : A → [0, 1] and NEx_1 : A → [0, 1] denotes the existence degree, uncertain, and non-existence of the element $a_i \in$ A respectively and

$0 \le Ex_1^2(a_i) + Un_1^2(a_i) + NEx_1^2(a_i) \le 2$

for every $a_i \in A, (i = 1,2,\cdots,n)$,

(ii)$E \subseteq A \times A$ where $Ex_2: A \times A \to [0,1], Un_2: A \times A \to [0,1]$ and $NEx_2: A \times A \to [0,1]$ are such that

$Ex_2(a_i, a_j) \ge \max\{Ex_1(a_i), Ex_1(a_j)\}$,

$Un_2(a_i, a_j) \ge \max\{Un_1(a_i), Un_1(a_j)\}$,

$NEx_2(a_i, a_j) \ge \min\{NEx_1(a_i), NEx_1(a_j)\}$

$0 \le Ex_2^2(a_i, a_j) + Un_2^2(a_i, a_j) + NEx_2^2(a_i, a_j) \le 2$

for every $(a_i, a_j) \in B, (i, j = 1,2,\cdots,n)$.

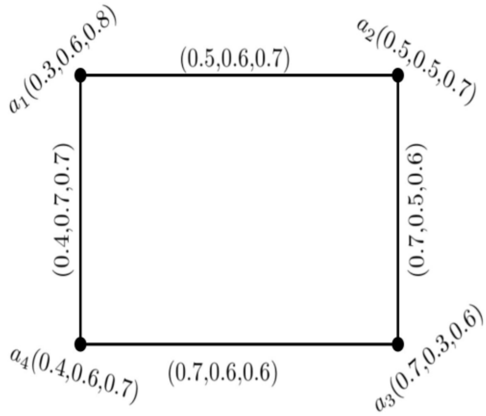

FIGURE 1. PCNG $Gr_\star\langle A, B\rangle$.

Definition 2.2

A PCNG $H_\star(A', B')$ is a Pythagorean Co-Neutrosophic subgraph (PCNSG) of $Gr_\star\langle A, B\rangle$ if $A' \subseteq A, B' \subseteq B$ such that $Ex'_{1i} \le Ex_{1i}, Un'_{1i} \le Un_{1i}, NEx'_{1i} \ge NEx_{1i}$ and $Ex'_{2ij} \le Ex_{2ij}, Un'_{2ij} \le Un_{2ij}, NEx'_{2ij} \ge NEx_{2ij}$.

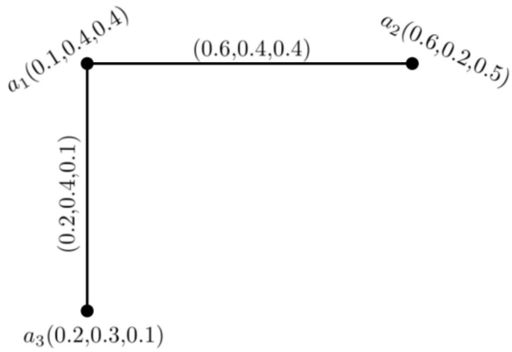

FIGURE 2. PCNSG of $Gr_\star\langle A, B\rangle$.

Definition 2.3

A PCNSG $H_\star(A', B')$ is called a spanning PCNSG of $Gr_\star\langle A, B\rangle$ if

(i) $A' = A, B' = B$

(ii) $Ex'_{1i} = Ex_{1i}, Un'_{1i} = Un_{1i}, NEx'_{1i} = NEx_{1i}$,

for all i, j.

Definition 2.4

Let $Gr_\star = \langle A, B\rangle$ be a PCNG. Then vertex set cardinality of A is defined by the equation (1),

$$|A| = \sum_{a_i \in A} \left(\frac{1 + Ex_1(a_i) + Un_1(a_i) - NEx_1(a_i)}{2}\right) \quad (1)$$

Definition 2.5

Let $Gr_\star = \langle A, B\rangle$ be a PCNG. Then edge set cardinality of B is defined by the equation (2),

$$|B| = \sum_{(a_i,a_j)\in B}\left(\frac{1+Ex_2(a_i,a_j)+Un_2(a_i,a_j)-NEx_2(a_i,a_j)}{2}\right) \quad (2)$$

Definition 2.6

Let $G_\star = \langle A, B\rangle$ be a PCNG. Then cardinality of $G_\star$ is defined by equation (3),

$$|G_\star| = \| A| + |B \|$$
$$=\left|\sum_{a_i\in A}\left(\frac{1+Ex_1(a_i)+Un_1(a_i)-NEx_1(a_i)}{2}\right) + \sum_{(a_i,a_j)\in B}\left(\frac{1+Ex_2(a_i,a_j)+Un_2(a_i,a_j)-NEx_2(a_i,a_j)}{2}\right)\right| \quad (3)$$

Definition 2.7

Let $Gr_\star = \langle A, B\rangle$ be a PCNG. The degree (Ex, Un, NEx) of a vertex a is nothing but the summation of values of each membership edge that are joining to a, which is denoted as $d_{Gr_\star}(a)$ and given by equation (4),

$$d_{Gr_\star}(a) = \left(d_{Ex}(a), d_{Un}(a), d_{NEx}(a)\right), \quad (4)$$

where $d_{Ex}(a) = \sum_{b\neq a} Ex_2(a,b)$,
$d_{Un}(a) = \sum_{b\neq a} Un_2(a,b)$ and
$d_{NEx}(a) = \sum_{b\neq a} NEx_2(a,b)$.

Definition 2.8

The minimum degree (Ex, Un, Nex) of a PCNG $Gr_\star = \langle A, B\rangle$ is denoted by (5),

$$\delta(Gr_\star) = (\delta_{Ex}(Gr_\star), \delta_{Un}(Gr_\star), \delta_{NEx}(Gr_\star)), \quad (5)$$

where $\delta_{Ex}(Gr_\star) = \min\{d_{Ex}(b)/b \in B\}$, $\delta_{Un}(Gr_\star) = \min\{d_{Un}(b)/b \in B$ and $\delta_{NEx}(Gr_\star) = \min\{d_{NEx}(b)/b \in B\}$.

Definition 2.9

The maximum degree (Ex, Un, NEx) of a PCNG$Gr_\star = \langle A, B\rangle$ is denoted by (6),

$$\Delta(Gr_\star) = \left(\Delta_{Ex}(Gr_\star), \Delta_{Un}(Gr_\star), \Delta_{NEx}(Gr_\star)\right), \quad (6)$$

where, $\Delta_{Ex}(Gr_\star) = \max\{d_{Ex}(b)/b \in B\}$, $\Delta_{Un}(Gr_\star) = \max\{d_{Un}(b)/b \in B\}$ and $\Delta_{NEx}(Gr_\star) = \max\{d_{NEx}(b)/b \in B\}$.

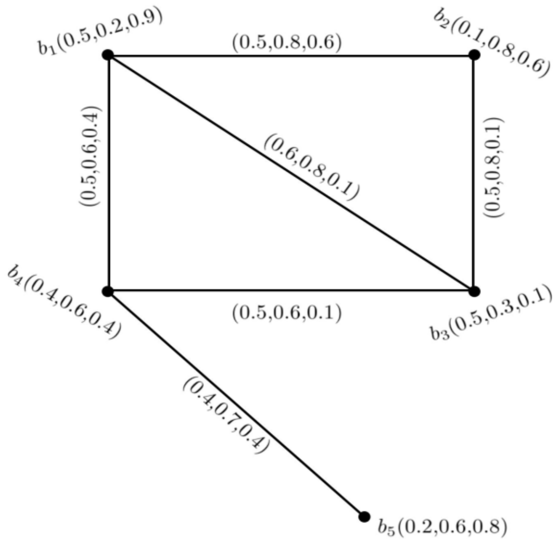

FIGURE 3. PCNG $Gr_\star\langle A, B\rangle$.

Example 2.10

Consider the PCNG of Fig.3

Vertex set cardinality of A is denoted by (7),

$$|A| = \sum_{a_i\in A}\left(\frac{1+Ex_1(a_i)+Un_1(a_i)-NEx_1(a_i)}{2}\right) \quad (7)$$

By Fig. 3, we get,

$$= \left(\frac{\big(1+(0.5+0.2-0.9)\big)+\big(1+(0.1+0.8-0.6)\big)+\big(1+(0.4+0.6-0.4)\big)+\big(1+(0.5+0.3-0.1)\big)+(1+(0.2+0.6-0.8))}{2}\right)$$

$$= \left(\frac{\big(1+(0.7-0.9)\big)+\big(1+(0.9-0.6)\big)+\big(1+(1.0-0.4)\big)+\big(1+(0.8-0.1)\big)+\big(1+(0.8-0.8)\big)}{2}\right)$$

$$= \left(\frac{(1-0.2)+(1+0.3)+(1+0.6)+(1+0.7)+(1+0)}{2}\right)$$

$$= \left(\frac{(0.8+1.3+1.6+1.7+1)}{2}\right)$$

$$= \left(\frac{6.4}{2}\right)$$

$$|A| = 3.2$$

The edge set cardinality of B is given by the following equation (8),

$$|B| = \sum_{(a_i,a_j)\in B}\left(\frac{1+Ex_2(a_i,a_j)+Un_2(a_i,a_j)-NEx_2(a_i,a_j)}{2}\right) \quad (8)$$

By Fig. 3, we get,

$$= \frac{\begin{array}{c}(1+(0.5+0.8-0.6))+(1+(0.5+0.8-0.1))+\\(1+(0.5+0.6-0.4))+(1+(0.5+0.6-0.1))+\\(1+(0.4+0.7-0.4))+(1+(0.6+0.3-0.1))\end{array}}{2}$$

$$= \frac{\begin{array}{c}(1+(1.3-0.6))+(1+(1.3-0.1))+\\(1+(1.1-0.4))+(1+(1.1-0.1))+\\(1+(1.1-0.4))+(1+(0.9-0.1))\end{array}}{2}$$

$$= \frac{\begin{array}{c}(1+0.7)+(1+1.2)+(1+0.7)+\\(1+1.0)+(1+0.7)+(1+0.8)\end{array}}{2}$$

$$= \frac{1.7+2.2+1.7+2.0+1.7+1.8}{2}$$

$$= \frac{11.1}{2}$$

$|B| = 5.55$

Cardinality set of $Gr_\star$ is $|Gr_\star| = \| A| + |B \| = 8.75$. Now, degree (Ex, Un, NEx) of vertices a_i are $d_{G_\star}(a_1) = (1.6,2.2,1.1)$, $d_{G_\star}(a_2) = (1.0,1.6,0.7)$, $d_{G_\star}(a_3) = (1.6,2.2,0.3)$, $d_{G_\star}(a_4) = (1.4,1.9,0.9)$, $d_{G_\star}(a_5) = (0.4,0.7,0.4)$.

Thus, the minimum degree (Ex, Un, NEx) of $Gr_\star$ is denoted by (9),

$$\delta(Gr_\star) = (\delta_{Ex}(Gr_\star), \delta_{Un}(Gr_\star), \delta_{NEx}(Gr_\star)) \quad (9)$$
$$= (0.4,0.7,0.4)$$

Maximum $(Ex, Un, NEx) -$ degree of $Gr_\star$ is denoted by (10),

$$\Delta(Gr_\star) = (\Delta_{Ex}(Gr_\star), \Delta_{Un}(Gr_\star), \Delta_{NEx}(Gr_\star)) \quad (10)$$
$$= (1.6,2.2,1.1)$$

Definition 2.11

An edge $e = (a, b)$ of PCNG $Gr_\star = \langle A, B \rangle$ is said to be an effective edge if (11),(12) and (13) are true.

$$Ex_2(a,b) = max\{Ex_1(a), Ex_1(b)\}, \quad (11)$$
$$Un_2(a,b) = max\{Un_1(a), Un_1(b)\} \quad (12)$$
$$NEx_2(a,b) = min\{NEx_1(a), NEx_1(b)\}. \quad (13)$$

Definition 2.12

A PCNG $Gr_\star = \langle A, B \rangle$ is known as complete if (14),(15) and (16) are true.

$$Ex_{2ij} = \max\{Ex_{1i}, Ex_{1j}\}, \quad (14)$$
$$Un_{2ij} = \max\{Un_{1i}, Un_{1j}\} \quad (15)$$
$$NEx_{2ij} = \min\{NEx_{1i}, NEx_{1j}\}, \text{ for all } a_i, a_j \in A. \quad (16)$$

Example 2.13

The graphical figure 4 is a complete PCNG.

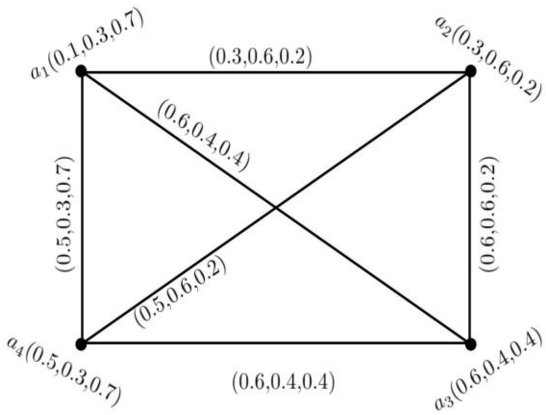

FIGURE 4. Complete PCNG of $Gr_\star\langle A, B \rangle$.

Definition 2.14

A PCNG $Gr_\star\langle A, B \rangle$ is called strong if (17),(18) and (19) are true.

$$Ex_{2ij} = \max\{Ex_{1i}, Ex_{1j}\}, \quad (17)$$
$$Un_{2ij} = \max\{Un_{1i}, Un_{1j}\}, \quad (18)$$
$$NEx_{2ij} = \min\{NEx_{1i}, NEx_{1j}\}, \quad (19)$$

for every $(a_i, a_j) \in B$.

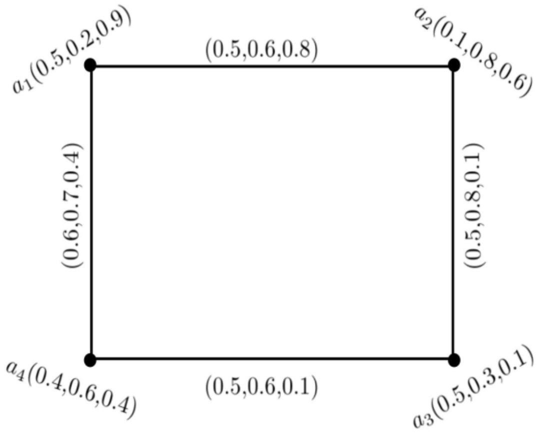

FIGURE 5. Strong PCNG $Gr_\star\langle A, B \rangle$.

Definition 2.15

A PCNG $Gr_\star\langle A, B \rangle$ is known to be regular (K_1, K_2, K_3) if $d_{Gr_\star}(a_i) = (K_1, K_2, K_3), \forall a_i \in A$ and also $Gr_\star$ is said to be a regular PCNG of (Ex, Un, NEx)-degree (K_1, K_2, K_3), where $K_1, K_2 \& K_3$ are real constants.

Example 2.16

The figure 6 is an example for (1.3,0.9,0.6)-regular PCNG.

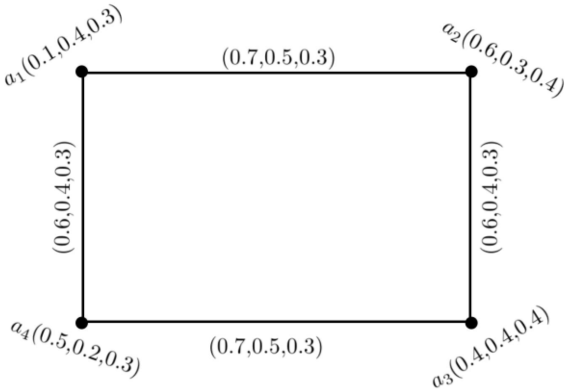

FIGURE 6. Regular PCNG $Gr_\star\langle A,B\rangle$.

III. COMPLEMENT OF STRONG PYTHAGOREAN CO-NEUTROSOPHIC GRAPH

Definition 3.1

The complement of a strong Pythagorean Co-Neutrosophic subgraph (PCNSG) $\mathrm{Gr}_\star = \langle \mathrm{A},\mathrm{B}\rangle$ is also strong PCNG. $\overline{\mathrm{Gr}_\star} = \langle \overline{\mathrm{A}},\overline{\mathrm{B}}\rangle$, where

(i) $\overline{A} = A$

(ii) $\overline{Ex_{11}} = Ex_{1i}, \overline{Un_{11}} = Un_{1i}$ and $\overline{NEx_{11}} = NEx_{1i}, \quad \forall\ i = 1,2,3,\cdots,n$

(iii) $$\overline{Ex_{21}} = \begin{cases} 0, & if \quad Ex_{2ij} > 0 \\ \max\{Ex_{1i}, Ex_{1j}\}, & if \quad Ex_{2ij} = 0 \end{cases}$$

$$= \max\{Ex_{1i}, Ex_{1j}\} - Ex_{2ij},$$

$for\ all\ i,j = 1,2,\cdots,n$

$$\overline{Un_{21}} = \begin{cases} 0, & if \quad Un_{2ij} > 0 \\ \max\{Un_{1i}, Un_{1j}\}, & if \quad Un_{2ij} = 0 \end{cases}$$

$$= \max\{Un_{1i}, Un_{1j}\} - Un_{2ij},$$

$for\ all\ i,j = 1,2,\cdots,n$

$$\overline{NEx_{21}} = \begin{cases} 0, & if \quad NEx_{2ij} > 0 \\ \min\{NEx_{1i}, NEx_{1j}\}, & if \quad NEx_{2ij} = 0 \end{cases}$$

$$= \min\{NEx_{1i}, NEx_{1j}\} - NEx_{2ij},$$

$for\ all\ i,j = 1,2,\cdots,n$

Example 3.2

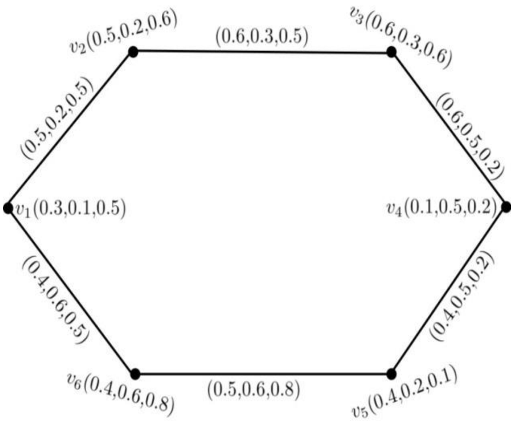

FIGURE 7. Strong PCNG $\mathrm{Gr}_\star\langle \mathrm{A},\mathrm{B}\rangle$.

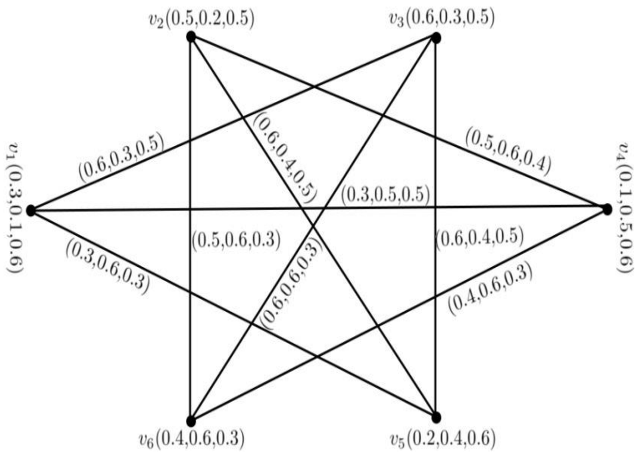

FIGURE 8. Complement of Strong PCNG $\overline{Gr_\star}$

Theorem 3.3

Let $\mathrm{Gr}_\star = \langle \mathrm{A},\mathrm{B}\rangle$ be a complete PCNG and its complement $\overline{\mathrm{Gr}_\star} = \langle \overline{\mathrm{A}},\overline{\mathrm{B}}\rangle$ then, $\overline{\mathrm{B}} = \varphi$.

Proof.

Let $Gr_\star = \langle A,B\rangle$ be a complete PCNG. Thus,

$$\begin{aligned} Ex_2(a,b) &= \max\{Ex_1(a), Ex_1(b)\} \\ Un_2(a,b) &= \max\{Un_1(a), Un_1(b)\} \\ NEx_2(a,b) &= \min\{NEx_1(a), NEx_1(b)\}, \end{aligned}$$

$for\ all\ a,b \in A.$

Assume the complement $\overline{Gr_\star} = \langle \overline{A},\overline{B}\rangle$ of $Gr_\star$. So,

$\overline{Ex_1}(a) = Ex_1(a);$

$\overline{Un_1}(a) = Un_1(a);$

$\overline{NEx_1}(a) = NEx_1(a)$, for all $a \in A$.

Therefore, for every $a \in A$,

$$\begin{aligned} \overline{Ex_2}(a,b) &= \max\{Ex_1(a), Ex_1(b)\} - Ex_2(a,b) \\ &= \max\{Ex_1(a), Ex_1(b)\} - \max\{Ex_1(a), Ex_1(b)\} \\ &= 0 \end{aligned}$$

$\overline{Un_2}(a,b) = \max\{Un_1(a), Un_1(b)\} - Un_2(a,b)$

$= \max\{Un_1(a), Un_1(b)\} - \max\{Un_1(a), Un_1(b)\}$
$= 0$

$\overline{NEx_2}(a,b) = \min\{NEx_1(a), NEx_1(b)\} - NEx_2(a,b)$
$= \min\{NEx_1(a), NEx_1(b)\} - \min\{NEx_1(a), NEx_1(b)\}$
$= 0$

So, $\left(\overline{Ex_2}(a,b), \overline{Un_2}(a,b), \overline{N}Ex_2(a,b)\right) = (0,0,0)$,
$for\ every\ a, b \in A$.

Thus, there don't exist arcs between any distinct vertices a and b of $\overline{Gr_\star}$. Therefore, $\overline{E} = \varphi$.

Remark 3.4
The complement of strong PCNG is again a strong PCNG.

Example 3.5

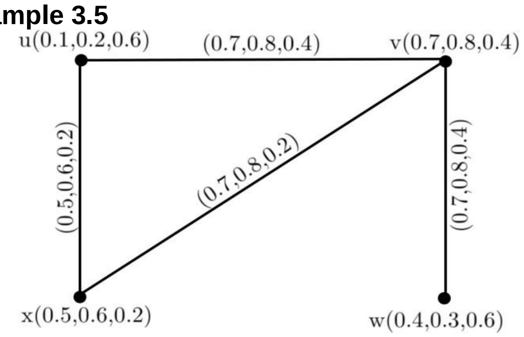

FIGURE 9. Strong PCNG $Gr_\star$

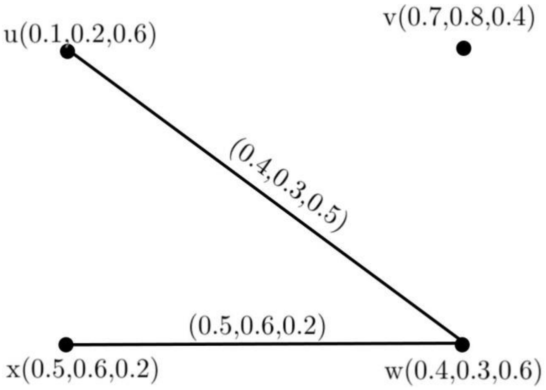

FIGURE 10. Complement of Strong PCNG $\overline{Gr_\star}$

Theorem 3.6
The complement of a strong PCNG $Gr_\star = \langle A, B\rangle$ yields the identical PCNG $Gr_\star$ itself. ($\overline{Gr_\star} = Gr_\star$)

Proof. Assume a strong PCNG $Gr_\star = \langle A, B\rangle$. So,

$Ex_2(a,b) = \max\{Ex_1(a), Ex_1(b)\}$
$Un_2(a,b) = \max\{Un_1(a), Un_1(b)\}$
$NEx_2(a,b) = \min\{NEx_1(a), NEx_1(b)\}$,

$for\ all\ (a,b) \in B$.

Consider the complement $\overline{Gr_\star} = \langle \overline{A}, \overline{B}\rangle$ of $Gr_\star$. So,

$\overline{Ex_1}(a) = Ex_1(a)$;
$\overline{Un_1}(a) = Un_1(a)$;
$\overline{NEx_1}(a) = NEx_1(u)$, $\forall a \in A$

$\overline{Ex_2}(a,b) = \max\{Ex_1(a), Ex_1(v)\} - Ex_2(a,b)$
$\overline{Un_2}(a,b) = \max\{Un_1(a), Un_1(v)\} - Un_2(a,b)$
$\overline{NEx_2}(a,b) = \min\{NEx_1(a), NEx_1(v)\} - NEx_2(a,b)$,

$for\ all\ (a,b) \in B$.

But in complement $\overline{\overline{Gr_\star}} = \langle \overline{\overline{A}}, \overline{\overline{B}}\rangle$,

$\overline{\overline{Ex_1}}(a) = \overline{Ex_1}(a) = Ex_1(a)$;
$\overline{\overline{Un_1}}(a) = \overline{Un_1}(a) = Un_1(a)$;
$\overline{\overline{NEx_1}}(a) = \overline{NEx_1}(a) = NEx_1(a),\ for\ every\ a \in A$

$\overline{\overline{Ex_2}}(a,b) = \max\{\overline{Ex_1}(a), \overline{Ex_1}(b)\} - \overline{Ex_2}(a,b)$
$= \max\{\overline{Ex_1}(a), \overline{Ex_1}(b)\} -$
$[\max\{Ex_1(a), Ex_1(b)\} - Ex_2(a,b)]$
$= \max\{Ex_1(a), Ex_1(b)\} -$
$\max\{Ex_1(a), Ex_1(b)\} + Ex_2(a,b)$
$= Ex_2(a,b)$

$\overline{\overline{Un_2}}(a,b) = \max\{\overline{Un_1}(a), \overline{Un_1}(b)\} - \overline{Un_2}(a,b)$
$= \max\{\overline{Un_1}(a), \overline{Un_1}(b)\} -$
$[\max\{Un_1(a), Un_1(b)\} - Un_2(a,b)]$
$= \max\{Un_1(a), Un_1(b)\} -$
$\max\{Un_1(a), Un_1(b)\} + Un_2(a,b)$
$= Un_2(a,b)$

$\overline{\overline{NEx_2}}(a,b) = \min\{\overline{NEx_1}(a), \overline{Ex_1}(b)\} - \overline{NEx_2}(a,b)$
$= \min\{\overline{NEx_1}(a), \overline{NEx_1}(b)\} -$
$[\min\{NEx_1(a), NEx_1(b)\} - NEx_2(a,b)]$
$= \min\{NEx_1(a), NEx_1(b)\} -$
$\min\{NEx_1(a), NEx_1(b)\} + NEx_2(a,b)$
$= NEx_2(a,b)$

Hence we conclude, $\overline{\overline{Gr_\star}} = Gr_\star$.

Theorem 3.7
Consider $Gr_\star = \langle A, B\rangle$ to be a strong PCNG and $\overline{Gr_\star} = \langle \overline{A}, \overline{B}\rangle$ be its complement strong PCNG. Then, the co-union $Gr_\star \cup \overline{Gr_\star}$ is always a complete PCNG.

Proof. Let $Gr_\star = \langle A, B\rangle$ be a strong PCNG. Then,

$Ex_2(a,b) = \max\{Ex_1(a), Ex_1(b)\}$
$Un_2(a,b) = \max\{Un_1(a), Un_1(b)\}$
$NEx_2(a,b) = \min\{NEx_1(a), NEx_1(b)\}$,

$for\ all\ (a,b) \in B$.

Assume the complement strong PCNG $\overline{Gr_\star} = \langle \overline{A}, \overline{B}\rangle$. So,

$\overline{Ex_{11}} = Ex_{1i}$;
$\overline{Un_{11}} = Un_{1i}$;
$\overline{NEx_{11}} = NEx_{1i}$,
$for\ every\ i = 1,2,3,\cdots,n$

$$\overline{Ex_{21}} = \max\{Ex_{1i}, Ex_{1j}\} - Ex_{2ij},$$
$$\overline{Un_{21}} = \max\{Un_{1i}, Un_{1j}\} - Un_{2ij},$$
$$\overline{NEx_{21}} = \min\{NEx_{1i}, NEx_{1j}\} - NEx_{2ij},$$

$for\ all\ i,j = 1,2,\cdots,n$

If $(a,b) \in B$,then

$\overline{Ex_2}(a,b) = \max\{Ex_1(u), Ex_1(v)\} - Ex_2(a,b)$
$= \max\{Ex_1(a), Ex_1(b)\} - \max\{Ex_1(a), Ex_1(b)\}$
$=$ 0 (Since, $Gr_\star$ is a strong co-neutrosophic graph)

$\overline{Un_2}(a,b) = \max\{Un_1(u), Un_1(v)\} - Un_2(a,b)$
$= \max\{Un_1(a), Un_1(b)\} - \max\{Un_1(a), Un_1(b)\}$
$=$ 0 (Since, $Gr_\star$ is a strong co-neutrosophic graph)

$\overline{NEx_2}(a,b) = \min\{NEx_1(u), NEx_1(v)\} - NEx_2(a,b)$
$= \min\{NEx_1(a), NEx_1(b)\} - \min\{NEx_1(a), NEx_1(b)\}$
$=$ 0 (Since, $Gr_\star$ is a strong co-neutrosophic graph)

As a result, for all $(a,b) \in B$ of $Gr_\star = \langle A, B\rangle$
$(\overline{Ex_2}(a,b), \overline{Un_2}(a,b), \overline{N}Ex_2(a,b)) = (0,0,0)\ in\ \overline{Gr_\star}$.

i.e., there won't exist an arc between vertices a and b in complement PCNG $\overline{Gr_\star}$. If $(a,b) \notin B$, then

$\overline{Ex_2}(a,b) = \max\{Ex_1(a), Ex_1(b)\} - Ex_2(a,b)$
$= \max\{Ex_1(a), Ex_1(b)\}$

$\overline{Un_2}(a,b) = \max\{Un_1(a), Un_1(b)\} - Un_2(a,b)$
$= \max\{Un_1(a), Un_1(b)\}$

$\overline{NEx_2}(a,b) = \min\{NEx_1(a), NEx_1(b)\} - NEx_2(a,b)$
$= \min\{NEx_1(a), NEx_1(b)\}$

Thus, for all $(a,b) \notin B$ of $Gr_\star = \langle A, B\rangle$,

$$\overline{Ex_2}(a,b) = \max\{Ex_1(a), Ex_1(b)\}$$
$$\overline{Un_2}(a,b) = \max\{Un_1(a), Un_1(b)\}$$
$$\overline{NEx_2}(a,b) = \min\{NEx_1(a), NEx_1(b)\},$$
$for\ all\ (a,b) \in \overline{B}\ in\ \overline{Gr_\star}$.

Thus, no incident vertices in $Gr_\star$ are incident vertices in $\overline{Gr_\star}$ and the arcs are effective arcs correspondingly.

If we find the co-union of $Gr_\star$ and $\overline{Gr_\star}$, the vertex set of $Gr_\star \cup \overline{Gr_\star}$, is the same as A itself with the same existence membership, uncertain and non-existence values as they are in $Gr_\star(\ or\ \ in\ \ \overline{Gr_\star}\ )$. The edge existence membership, uncertain and non-existence values of an arbitrary edge (a,b) in co-union $Gr_\star \cup \overline{Gr_\star}$ because

$$(Ex_2 \cup \overline{Ex_2})(a,b) = \begin{cases} Ex_2(a,b), & if\ (a,b) \in B\backslash\overline{B} \\ \overline{Ex_2}(a,b), & if\ (a,b) \in \overline{B}\backslash B. \end{cases}$$

$$(Un_2 \cup \overline{Un_2})(a,b) = \begin{cases} Un_2(a,b), & if\ (a,b) \in B\backslash\overline{B} \\ \overline{Un_2}(a,b), & if\ (a,b) \in \overline{B}\backslash B. \end{cases}$$

$$(NEx_2 \cup \overline{NEx_2})(a,b) = \begin{cases} NEx_2(a,b), & if\ (a,b) \in B\backslash\overline{B} \\ \overline{NEx_2}(a,b), & if\ (a,b) \in \overline{B}\backslash B. \end{cases}$$

Therefore, $Gr_\star \cup \overline{Gr_\star}$ produces its basal graph as a complete graph, where $Gr_\star$ and $\overline{Gr_\star}$ are strong PCNG.

Since the co-union of strong PCNGs is again a strong PCNG, $Gr_\star \cup \overline{Gr_\star}$ transforms to a strong PCNG. Therefore, $Gr_\star \cup \overline{Gr_\star}$ is a complete PCNG.

IV. CO-COMPLEMENT OF PYTHAGOREAN CO-NEUTROSOPHIC GRAPHS

Definition 4.1

The Co-complement of a Pythagorean Co-Neutrosophic graph(PCNG) $Gr_\star = \langle A, B\rangle$ is a graph $\widetilde{Gr_\star} = (\widetilde{A}, \widetilde{B})$ where

(i) $\tilde{A} = A$

(ii) $\widetilde{Ex_{11}} = Ex_{1i}$,
$\widetilde{Un_{11}} = Un_{1i}\ and$
$\widetilde{NEx_{11}} = NEx_{1i},\ \forall\ i = 1,2,3,\dots,n$

(iii) $\widetilde{Ex_{21j}} = 1 - Ex_{2ij} + \max\{Ex_{1i}, Ex_{1j}\}$,
$\widetilde{Un_{21j}} = 1 - Un_{2ij} + \max\{Un_{1i}, Un_{1j}\}\ and$
$\widetilde{NEx_{21j}} = 1 - NEx_{2ij} + \min\{NEx_{1i}, NEx_{1j}\}$,
$\forall (a_i, a_j) \in B$.

Example 4.2

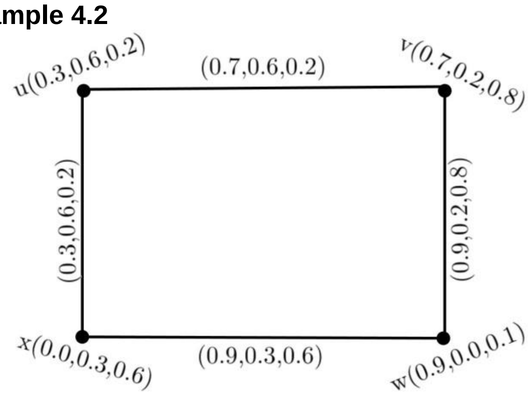

FIGURE 11. PCNG $Gr_\star$

Proposition 4.3

The co-complement of a PCNG need not be again a PCNG.

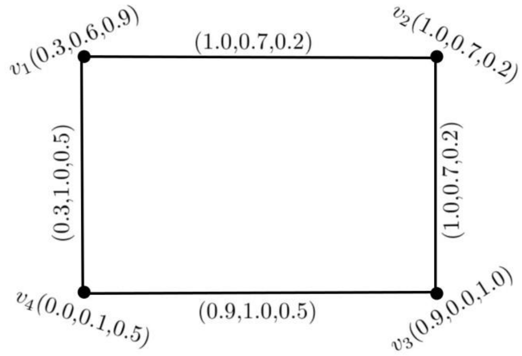

FIGURE 12. Co-complement PCNG ($\widetilde{Gr_\star}$).

Theorem 4.4
Consider a complete PCNG $\mathrm{Gr}_\star$. Therefore, co-complement $\widetilde{\mathrm{Gr}_\star}$ is $(\mathrm{n}-1, \mathrm{n}-1, \mathrm{n}-1)$-regular.

Proof. Let $Gr_\star = \langle A, B\rangle$ be a complete PCNG.
$\therefore Ex_{2ij} = \max\{Ex_{1i}, Ex_{1j}\}$,
$Un_{2ij} = \max\{Un_{1i}, Un_{1j}\}$ *and*
$NEx_{2ij} = \min\{NEx_{1i}, NEx_{1j}\}$,
for all $i = 1,2,\dots,n$.

But for the co-complement $\widetilde{Gr_\star}$,
$\widetilde{Ex_{21}} = 1 - Ex_{2ij} + \max\{Ex_{1i}, Ex_{1j}\} = 1 - Ex_{2ij} + Ex_{2ij} = 1$,
for all $i, j = 1,2,3,\cdots,n$.

Now,
$\widetilde{Un_{21}} = 1 - Un_{2ij} + \max\{Un_{1i}, Un_{1j}\} = 1 - Un_{2ij} + Un_{2ij} = 1$,
for all $i, j = 1,2,3,\cdots,n$.

$\widetilde{NEx_{21}} = 1 - NEx_{2ij} + \min\{NEx_{1i}, NEx_{1j}\} = 1 - NEx_{2ij} + NEx_{2ij} = 1$, for all $i, j = 1,2,3,\cdots,n$.

As the basal graph of $Gr_\star$ is complete,

$$d_{Gr_\star}(a_i) = \left(\sum_{k=1}^{n=1} 1, \sum_{k=1}^{n=1} 1, \sum_{k=1}^{n=1} 1\right)$$

$$= (n-1, n-1, n-1)$$

Thus, $\widetilde{Gr_\star}$ *is* $(n-1, n-1, n-1)$-regular.

Theorem 4.5
Let $\mathrm{Gr}_\star = \langle \mathrm{A}, \mathrm{B}\rangle$ be a PCNG. Thus, the basal graph of $\widetilde{\mathrm{Gr}_\star}$ is complete.

Proof.
Let $\mathrm{Gr}_\star\langle \mathrm{A}, \mathrm{B}\rangle$ be a PCNG co neutrosophic graph *and* $\widetilde{Gr_\star} = (\tilde{V}, \tilde{E})$ be the complement of $Gr_\star$.

We have,

(i) $\tilde{A} = A$

(ii) $\widetilde{Ex_{11}} = Ex_{1i}$,
$\widetilde{Un_{11}} = Un_{1i}$ *and*
$\widetilde{NEx_{11}} = NEx_{1i}, \forall i = 1,2,3,\cdots,n$

(iii) $\widetilde{Ex_{21}} = 1 - Ex_{2ij} + \max\{Ex_{1i}, Ex_{1j}\}$,
$\widetilde{Un_{21}} = 1 - Un_{2ij} + \max\{Un_{1i}, Un_{1j}\}$ *and*
$\widetilde{NEx_{21}} = 1 - NEx_{2ij} + \min\{NEx_{1i}, NEx_{1j}\}$,
forall $(a_i, a_j) \in B$.

If $Ex_{2ij} \neq 0, Ex_{2ij} > 0,$ *forall* $(a_i, a_j) \in B \Rightarrow \widetilde{Ex_{21}} > 0$

If $Ex_{2ij} = 0, \widetilde{Ex_{21}} = 1 + \max\{Ex_{1i}, Ex_{1j}\} > 0$

Therefore $\widetilde{Ex_{21}} > 0,$ *for all* $i = 1,2,3,\cdots,n$

If $Un_{2ij} \neq 0, Un_{2ij} > 0,$ *forall* $(a_i, a_j) \in E \Rightarrow \widetilde{Un_{21}} > 0$

If $Un_{2ij} = 0, \widetilde{Un_{21}} = 1 + \max\{Un_{1i}, Un_{1j}\} > 0$

Therefore $\widetilde{Un_{21}} > 0,$ *for all* $i = 1,2,3,\cdots,n$

If $NEx_{2ij} \neq 0, NEx_{2ij} > 0, \forall (a_i, a_j) \in E \Rightarrow \widetilde{NEx_{21}} > 0$

If $NEx_{2ij} = 0, \widetilde{NEx_{2ij}} = 1 + \min\{NEx_{1i}, NEx_{1j}\} > 0$

Therefore, $\widetilde{NEx_{21}} > 0,$ *forall* $i = 1,2,3,\cdots,n$

This implies that the basal graph of $\widetilde{Gr_\star}$ is complete.

Theorem 4.6
Consider a PCNG $Gr_\star = \langle A, B\rangle$. Thus, $\widetilde{\widetilde{Gr_\star}} = Gr_\star$.

Proof. Consider the co-complement $\widetilde{Gr_\star} = (\tilde{A}, \tilde{B})$ of PCNG $Gr_\star = \langle A, B\rangle$. We have,
(i) $\tilde{A} = A$

(ii) $\widetilde{Ex_{11}} = Ex_{1i}$,
$\widetilde{Un_{11}} = Un_{1i}$ *and*
$\widetilde{NEx_{11}} = NEx_{1i}, \forall i = 1,2,3,\cdots,n$

(iii) $\widetilde{Ex_{21}} = 1 - Ex_{2ij} + \max\{Ex_{1i}, Ex_{1j}\}$,
$\widetilde{Un_{21}} = 1 - Un_{2ij} + \max\{Un_{1i}, Un_{1j}\}$ *and*
$\widetilde{NEx_{21}} = 1 - NEx_{2ij} + \min\{NEx_{1i}, NEx_{1j}\}$,
$\forall (a_1, a_j) \in B$.

Take the co-complement $\widetilde{\widetilde{Gr_\star}} = \langle \tilde{\tilde{A}}, \tilde{\tilde{B}}\rangle$ of $\widetilde{Gr_\star} = (\tilde{A}, \tilde{B})$. As a result,
(i) $\tilde{\tilde{A}} = \tilde{A} = A$

(ii) $\widetilde{\widetilde{Ex_{11}}} = \widetilde{Ex_{11}} = Ex_{1i}$,
$\widetilde{\widetilde{Un_{11}}} = \widetilde{Un_{11}} = Un_{1i}$ and
$\widetilde{\widetilde{NEx_{11}}} = \widetilde{NEx_{11}} = NEx_{1i}, \forall i = 1,2,3,\cdots,n$

(iii)

$\widetilde{\widetilde{Ex_{21}}} = 1 - \widetilde{Ex_{21}} + \max\{\widetilde{Ex_{11}}, \widetilde{Ex_{1j}}\}$
$= 1 - [1 - Ex_{2ij} + \max\{Ex_{1i}, Ex_{1j}\}] + \max\{\widetilde{Ex_{11}}, \widetilde{Ex_{1j}}\}$
$= 1 - [1 - Ex_{2ij} + \max\{Ex_{1i}, Ex_{1j}\}] + max\{Ex_{1i}, Ex_{1j}\}$
$= Ex_{2ij}, \quad \forall (a_i, a_j) \in B$

$\widetilde{Un_{21}} = 1 - \widetilde{Un_{21}} + \max\{\widetilde{Un_{11}}, \widetilde{Un_{1j}}\}$
$= 1 - [1 - Un_{2ij} + \max\{Un_{1i}, Un_{1j}\}]$
$+\max\{\widetilde{Un_{11}}, \widetilde{Un_{1j}}\}$
$= 1 - [1 - Un_{2ij} + \max\{Un_{1i}, Un_{1j}\}]$
$+max\{Un_{1i}, Un_{1j}\}$
$= Unx_{2ij}, \quad \forall\ (a_i, a_j) \in B$

$\widetilde{NEx_{21}} = 1 - \widetilde{NEx_{21}} + \min\{\widetilde{NEx_{11}}, \widetilde{NEx_{1j}}\}$
$= 1 - [1 - NEx_{2ij} + \min\{NEx_{1i}, NEx_{1j}\}]$
$+\min\{\widetilde{NEx_{11}}, \widetilde{NEx_{1j}}\}$
$= 1 - [1 - NEx_{2ij} + \min\{NEx_{1i}, NEx_{1j}\}]$
$+min\{NEx_{1i}, NEx_{1j}\}$
$= NEx_{2ij}, \quad \forall\ (a_i, a_j) \in B$

Definition 4.7
A PCNG $Gr_\star$ is self-co-complementary when $Gr_\star = \widetilde{Gr_\star}$.

Example 4.8

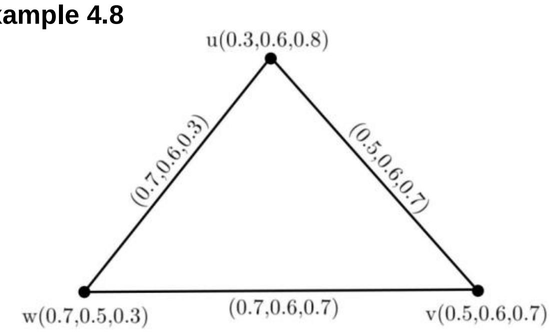

FIGURE 13. PCNG $Gr_\star$.

Theorem 4.9
Consider a PCNG $Gr_\star = \langle A, B\rangle$. Assume $Gr_\star$ is self-co-complementary then,

$$Ex_{2ij} = \frac{1}{2}[1 + \max\{Ex_{1i}, Ex_{1j}\}]$$
$$Un_{2ij} = \frac{1}{2}[1 + \max\{Un_{1i}, Un_{1j}\}]$$
$$NEx_{2ij} = \frac{1}{2}[1 + \min\{NEx_{1i}, NEx_{1j}\}].$$

Proof.
Let us take a self-co-complementary PCNG $Gr_\star = \langle A, B\rangle$, which results $Gr_\star = \widetilde{Gr_\star}$. Therefore, $(Ex_1, Un_1, NEx_1) = (\widetilde{Ex_1}, \widetilde{Un_1}, \widetilde{NEx_1})$ and $(Ex_2, Un_2, NEx_2) = (\widetilde{Ex_2}, \widetilde{Un_2}, \widetilde{NEx_2})$.
Thus

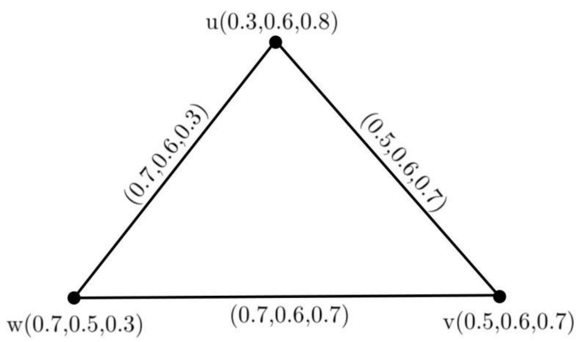

FIGURE 14. Co-complement PCNG $\widetilde{Gr_\star}$.

$\widetilde{Ex_{21}} = 1 - Ex_{2ij} + \{Ex_{1i}, Ex_{1j}\}$
$Ex_{2ij} = 1 - Ex_{2ij} + \max\{Ex_{1i}, Ex_{1j}\}$
$2Ex_{2ij} = 1 + \max\{Ex_{1i}, Ex_{1j}\}$
$Ex_{2ij} = \frac{1}{2}[1 + \max\{Ex_{1i}, Ex_{1j}\}]$

$\widetilde{Un_{21}} = 1 - Un_{2ij} + \max\{Un_{1i}, Un_{1j}\}$
$Un_{2ij} = 1 - Un_{2ij} + \max\{Un_{1i}, Un_{1j}\}$
$2Un_{2ij} = 1 + \max\{Un_{1i}, Un_{1j}\}$
$Un_{2ij} = \frac{1}{2}[1 + \max\{Un_{1i}, Un_{1j}\}]$

and

$\widetilde{NEx_{21}} = 1 - NEx_{2ij} + \min\{NEx_{1i}, NEx_{1j}\}$
$NEx_{2ij} = 1 - NEx_{2ij} + \min\{NEx_{1i}, NEx_{1j}\}$
$2NEx_{2ij} = 1 + \min\{NEx_{1i}, NEx_{1j}\}$
$NEx_{2ij} = \frac{1}{2}[1 + \min\{NEx_{1i}, NEx_{1j}\}]$

V. APPLICATION

In [11], an application was discussed regarding the COVID-19 spread in the case of an intuitionistic co-fuzzy graph. Here, an uncertain membership for vertices and edges can be defined to deal with the PCNGs. The uncertain membership of vertex and edge can be taken as an imbalanced health rate of individuals due to other health issues, and mediatory infection between persons due to contact, respectively. Through this approach, one can compare the different criteria of a pandemic spread. As a result, the affected and recovery rate of an individual or a community can be obtained.

VI. CONCLUSION

A research gap between intuitionistic and neutrosophic kind of graphical approaches has been filled by introducing complementation in Pythagorean Co-Neutrosophic graphs (PCNG). This paper deals with the PCNG and some cases related to PCNG like subgraph, vertex degree, and strong property. The complement of some types of PCNGs is discussed in detail. In addition, the co-complement property was applied to PCNGs. Through this work, the application related to the reverse kind of Pythagorean neutrosophic graphs can be studied further. The novelty of our work is enforced by the above new analyzing properties with PCNG and its application. In the future, we had planned to elaborate this co-behaviour property to other kinds of neutrosophic graphs and their properties will be studied briefly.

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