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Chaos Synchronization of the Lu System Using Single-Variable Feedback

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***Abstract*–This paper explores a simple yet effective way to synchronize the chaotic Lü system using just one variable from the master system. Rather than relying on full-state observation or advanced nonlinear control, the method uses a straightforward linear feedback approach and takes advantage of the inherent stability in cascade-connected systems to achieve synchronization.**

One of the main strengths of this approach is its efficiency. By transmitting only a single state variable, it keeps communication demands low—something that's especially helpful in real-time applications or when resources are limited. Another benefit is that the method doesn't depend on knowing the bounds of the master system's trajectories in advance, which makes it more flexible for systems that are unpredictable or constantly changing. The controller itself is also relatively simple to put into practice, avoiding the complexity often seen in other synchronization methods.

The approach is backed by solid theoretical analysis, and simulation results using MATLAB show that it works well in practice. Overall, this method offers a lightweight and practical solution for chaos synchronization—ideal for situations where minimal data and easy implementation are key.

***Keywords*—Lu System, Chaotic Dynamics, Synchronization, Single-Variable Feedback, Nonlinear Control.**

I. INTRODUCTION

In the last few decades, chaotic systems have gone from being mere mathematical curiosities to becoming valuable tools across various scientific and engineering domains. What's fascinating about chaos isn't just the unpredictability—but the fact that it comes from systems that are entirely deterministic. A tiny change at the beginning can lead to dramatically different outcomes. While this sensitivity was once seen as a problem, today it's a key feature in everything from secure communications and robotics to lasers and even medical devices [1–5].

But chaos is tricky. On one hand, it gives us rich, flexible behavior. On the other, it makes systems hard to manage. That's where synchronization comes in—getting one chaotic system to track or follow another. In cryptography, for instance, it's used to scramble and unscramble signals. In medicine, it can help regulate erratic heart rhythms [6].

Still, managing chaotic systems is anything but simple. They're nonlinear, highly sensitive, and often complex in dimension. Even small measurement errors or parameter mismatches can lead to failure. Over time,

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researchers have developed various tools: backstepping for layered stabilization [3], adaptive controls that adjust on the fly [4], impulsive controls for quick corrections [5], and sliding mode approaches for robust performance under disturbance [7].

Lately, hybrid control methods have caught attention. These combine different control techniques to balance out their strengths and weaknesses [8–10]. Think of it as building a balanced team—where backstepping brings structure, sliding mode adds robustness, and adaptive control makes the system more flexible. Such combinations have been promising, especially for real-world systems that rarely behave in nice, linear ways [11–22].

However, there's a common assumption in most of these methods: that we can observe and control all variables of the system. In real life, that's often not possible. Full-state measurement is expensive, and sometimes just impractical—especially in systems like the Lü system, which has multiple variables and complicated dynamics. That's what motivated this study.

Here, we present a different approach. Instead of watching everything, we use just one variable from the master system to synchronize the rest. This simple feedback strategy reduces the amount of data needed, simplifies the controller, and makes real-time applications more practical. Whether it's a drone with limited sensors or a small embedded medical device, needing only a single signal makes things easier.

This paper focuses on the Lü system, a classic three-variable chaotic system. We build a controller based on hybrid principles and prove that it can achieve synchronization using only one state variable. We also provide simulation results to show it works—not just in theory, but in practice too.

II. Chaos Synchronization of Lu Systems

Consider two distinct nonlinear systems:

$$\dot{x} = f(t, x), \tag{1}$$

$$\dot{y} = g(t, y) + u(t, x, y). \tag{2}$$

Where $x, y \in \Re^n, f, g \in C^r[\Re^+ \times \Re^n, \Re^n]$,

$u \in C^r[\Re^+ \times \Re^n \times \Re^n, \Re^n], r \geq 1, \Re^+$ is the collection of real numbers that are not negative. In this context, $\Re^n$ denotes the set of non-negative real values. Let system (1) represent the driving system and system (2) the response system, with $u(t, x, y)$ as the control input.

Definition 1. The response system and the drive system are considered synchronized if, for any given initial conditions, their trajectories converge over time $x(t_0), y(t_0) \in \Re^n$,

$$\lim_{t \to +\infty} \| x(t) - y(t) \| = 0.$$

In this section, we examine two Lu systems, where the system labeled with subscript 1 serves as the driving system and influences the response system labeled with subscript 2. The dynamics of both systems are described by the following equations:

$$\begin{aligned} \dot{x}_1 &= a(y_1 - x_1), \\ \dot{y}_1 &= -x_1 z_1 + c y_1, \\ \dot{z}_1 &= x_1 y_1 - b z_1. \end{aligned} \tag{3}$$

And

$$\begin{aligned} \dot{x}_2 &= a(y_2 - x_2) + u, \\ \dot{y}_2 &= -x_2 z_2 + c y_2, \\ \dot{z}_2 &= x_2 y_2 - b z_2. \end{aligned} \tag{4}$$

In equation (4), a control function is introduced, and our task is to determine its specific form. The error system is formulated as the difference between the drive system (3) and the controlled response system (4). To proceed, we define the state errors as the variation between the response system (4) and the drive system (3):

$$\begin{aligned} e_x &= x_2 - x_1, \\ e_y &= y_2 - y_1, \\ e_z &= z_2 - z_1. \end{aligned} \tag{5}$$

By taking the difference between equations (3) and (4) and applying the notation defined in equation (5), we obtain:

$$\begin{aligned} \dot{e}_x &= a(e_y - e_x) + u(t), \\ \dot{e}_y &= x_1 z_1 - x_2 z_2 - c e_y, \\ \dot{e}_z &= x_2 y_2 - x_1 y_1 - b e_z. \end{aligned} \tag{6}$$

We define the active control functions in the following manner:

$$u(t) = V_1(t) - a y_2 + a y_1.$$

As a result, the error system (6) is transformed into:

$$\begin{aligned} \dot{e}_x &= V_1(t) - a e_x, \\ \dot{e}_y &= V_2(t) - c e_y, \\ \dot{e}_z &= V_3(t) - b e_z. \end{aligned} \tag{7}$$

The error system (7) is a linear system where the control input depends on the error terms. Several control function choices are available, and we select:

$$\begin{bmatrix} V_1(t) \\ V_2(t) \\ V_3(t) \end{bmatrix} = A \begin{bmatrix} e_x \\ e_y \\ e_z \end{bmatrix}.$$

Where A is a fixed constant matrix. We select the matrix A in the following form:

$$A = \begin{pmatrix} -a & 0 & 0 \\ 0 & -c & 0 \\ 0 & 0 & -b \end{pmatrix}.$$

Given the eigenvalues of A are $-a,-b,-c,$ where a,b,c constitute positive constants. Consequently e_x, e_y, e_z approach zero as t tends to $+\infty$ Consequently, the Lu's systems (3) derived from (4) are synchronous.

III. Numerical Simulations

The fourth-order Runge-Kutta integration method is employed to solve two systems of differential equations (3) and (4) with a time step value of 0.01. The parameter values in (3) are selected as $a=1, b=3, c=2$ to ascertain the erratic dynamics of Lu's systems. The preliminary parameters of the driving system are $x_1(0)=-1,\ y_1(0)=1, z_1(0)=2$ the initial conditions of the response system are $x_2(0)=1,\ y_2(0)=2, z_2(0)=3.$ Consequently, the preliminary values of the error system are $e_x(0)=-1, e_y(0)=1, e_z(0)=2.$ Figure 1. Demonstrate the state faults (e_x, e_y, e_z) of Lu's systems of equations with the active control engaged. The slave trajectories align closely with the corresponding master trajectories. Within one second, the tracking error is below the level 10^{-3}.

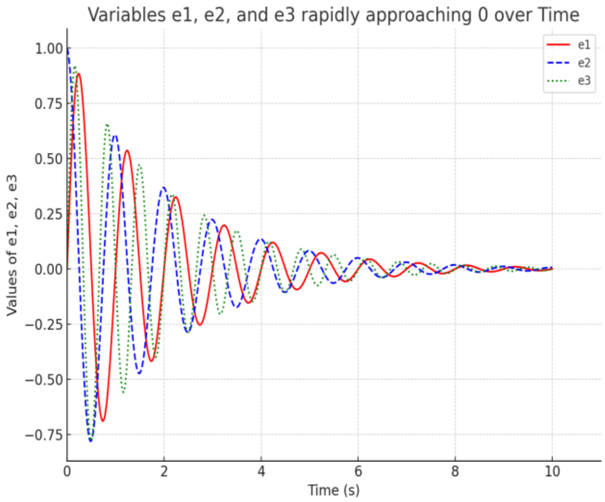

FIGURE 1. The temporal response of states for the master system (3) and the slave system (4) utilizing the control law (6).

IV. Conclusion

This paper introduces a streamlined yet powerful approach to synchronizing chaotic systems—focusing on the Lu system—using feedback from just a single variable of the master system. By applying stability theory for cascade-connected systems, we derive a sufficient condition that guarantees successful synchronization. The control strategy is intentionally kept simple: a linear feedback law that avoids the complexity of full-state observers and nonlinear controllers, yet delivers reliable performance.

A standout contribution of this work is its minimal data requirement. Instead of relying on full system measurements, our method synchronizes chaotic systems using just one transmitted signal. This dramatically reduces both computational overhead and sensor demands, making the method highly relevant for real-time applications like remote sensing, embedded control systems, and biomedical devices. The design also sidesteps the need to estimate or bound the master system's full trajectory, which is often a limiting factor in conventional approaches.

Beyond the Lu system, the proposed framework is adaptable to other high-dimensional chaotic systems such as Lorenz, Chen, and Chua systems—each widely used in both theoretical research and applied engineering. This generalizability makes the single-variable feedback approach a compelling tool for chaos synchronization across a broad range of use cases, including secure communication, industrial automation, and autonomous systems.

To back the theory, we present numerical simulations that validate the method's effectiveness under various initial conditions. The results show robust synchronization behavior, even when the initial mismatch between systems is large—underscoring the method's practicality in real-world settings.

This work opens up several promising avenues for future research. Extensions to more complex chaotic systems, implementation in real-time hardware, and integration with adaptive or learning-based controllers could further enhance performance and robustness—especially in systems with uncertain or time-varying dynamics.

In summary, this paper offers an efficient, low-data, and broadly applicable solution for chaos synchronization. The simplicity of the single-variable feedback method, paired with its solid theoretical grounding, makes it a strong candidate for deployment in real-time and resource-constrained environments—where complexity is costly and reliability is key.

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AUTHOR CONTRIBUTIONS

Grienggrai Rajchakit: Conceptualization, Data Curation, Methodology, Validation, Writing – Original Draft Preparation, Project Administration, Writing – Review & Editing.

CONFLICT OF INTERESTS

No conflict of interest was disclosed.

ETHICS STATEMENT

This research did not involve human participants, animal subjects, or sensitive personal data, and therefore did not require ethical approval.

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