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Some Insights on Pythagorean Neutrosophic Graphs

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***Abstract* – Pythagorean neutrosophic graphs (PNeuGr) are a specialized extension of the neutrosophic graphical idea, where the total sum range of memberships is adjusted by squaring each membership. This article is furnished to enhance the handling of uncertain events in a complex environment. The discussion encloses the irregular properties of the PNeuGr and its practical implications.**

Keywords— Pythagorean Neutrosophic Set, Pythagorean Neutrosophic Graph, Irregular, Edge Irregular, Neighborly Irregular, Highly Irregular.

TABLE 1. Abbreviations Used.

Description	Abbreviation
Fuzzy Set	FuS
Fuzzy Graph	FuG
Intuitionistic Fuzzy Set	IN-FuS
Intuitionistic Fuzzy Graph	IN-FuG
Neutrosophic Set	NeuS
Neutrosophic Graphs	NeuGr
Pythagorean Neutrosophic Graphs	PNeuGr
Neighborly	NeiG
Neighborhood	NEl
Irregular Pythagorean Neutrosophic Graphs	iPNeuGr
Highly	HiG
Strongly	Str

I. Introduction

Graph theory has replaced other application-oriented fields since it deals with the real picturization of events with the help of components such as vertices and edges. Euler sowed the seed for the evolution of graph theory and its structural discussions on real life through the famous bridge problem. Then, the crisp and integer-based graph theory helped the other researchers to imagine their problem with graphical properties. However, inaccurate information exists regarding the outcome. This problem was understood, and the development is intended to find the extensible set theory concept named "FuS" [1]. With this, the performance of the fuzzy graphical models is analyzed and portrayed [2]. Many insightful properties and developments in FuG theory have been flourished [3].

From one point of view, the results are still inadequate and incomplete. The recognition and restructuring of the existing set theory concept happened by adding a non-membership element. This improved set-theoretical approach was named "IN-FuS" [4]. This set theory acts as a base and implements a developed graphical structure with fuzzy value-based elements, which is declared in the name "IN-FuG" [5]. Further excavation of some essential

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results and theories on IN-FuG is then executed and claimed [6].

Later, an upgraded set concept evolved with an increased total sum range of memberships [7]. It is announced for a separate membership to organize the uncertain things of an event, and is named "indeterminacy." Based on this indeterminacy, the set theory is called "NeuS" [8], a primary requirement for developing NeuGr. Preliminary work on the NeuGr has been carried out by analyzing all basic terminologies and functions [9]. The reframed structure of this graph theory with restricted limits by implementing a standard interval range for each membership [10] is delivered and called "Single Valued NeuGr." Some essential properties, such as degree, size, and order, are portrayed in this graph.

Next to this, the Pythagorean fuzzy set [11] was introduced, and it replaced the IN-FuS since this allows a wide range of values for memberships as it is squared. The Pythagorean FuG [12] was coined based on this set concept. In the same way, Pythagorean NeuS [13] was used to expand the applicability of membership values; thereby, it is used to enforce the PNeuGr [14]. The generation of this graph type is used to learn the regularity properties of PNeuGr and its application [15]. Also, some PNeuGr operations with different products are carried out [16]. The NeiG edge irregularity property on interval-valued PNeuGr [17] is explored using other criteria. The product discussion was recently done on Pythagorean Co-NeuGr [18], and an application on brain network analysis was framed. The complement and anti-complement properties of Pythagorean Co-NeuGr [19] are listed with examples. At a glance at this research, the base works [20-21] are essential when applying the discussion in PNeuGr and its properties.

This article encompasses some essential & basic definitions and results on PNeuGr. The irregularity and edge-irregular properties of PNeuGr are demonstrated in a wide range. The sectional highlights are listed in the following way: Section I captures the introductory works on the FuS and its extensional concepts. Section II covers the basic terminologies of the neutrosophic set and graph. Also, the definition of PNeuGr and related terms is noted. The irregularity and edge irregularity of PNeuGr are elaborated in Section III with some theorem results. Section IV consolidates the final work on PNeuGr and our team's future work. An application regarding the proposed work is portrayed in Section V. Section VI encloses the concluding remarks and our future insight on PNeuGr.

II. Preliminary definitions

Definition 2.1. [7]

Consider Z to be the universal set. A NeuS $\overline{\mathrm{N}}$ framed on Z is called as $\overline{\mathrm{R}} = \{(t, \mathrm{T}_{\overline{N}}(t), \mathrm{I}_{\overline{N}}(t), \mathrm{F}_{\overline{N}}(t)) : t \in X\}$, where $\mathrm{T}_{\overline{N}}(t) : Z \to [0,1]$, $\mathrm{I}_{\overline{N}}(t) : Z \to [0,1]$, $\mathrm{F}_{\overline{N}}(t) : Z \to [0,1]$ are said to be functions for truth(available), indeterminacy(unsure) and false(unavailable) membership of t on $\overline{\mathrm{N}}$ respectively and it satisfies the condition $0 \le \mathrm{T}_{\overline{N}} + \mathrm{I}_{\overline{N}} + \mathrm{F}_{\overline{N}} \le 3, \forall\, t \in Z$.

Definition 2.2. [10]

A NeuGr is mentioned as $G = (\ddot{Y}, \alpha, \beta)$, where $\alpha = (\mathrm{T}_{\ddot{a}}, \mathrm{I}_{\ddot{a}}, \mathrm{F}_{\ddot{a}})$ and $\beta = (\mathrm{T}_{b}, \mathrm{I}_{b}, \mathrm{F}_{b})$ and holds the following conditions,

(i) Let $\mathrm{T}_{\ddot{a}} : \ddot{Y} \to [0,1]$, $\mathrm{I}_{\ddot{a}} : \ddot{Y} \to [0,1]$ and $\mathrm{F}_{\ddot{a}} : \ddot{Y} \to [0,1]$ denote the available, unsure, and unavailable memberships of the element $a_i \in \ddot{Y}$, respectively and $0 \le \mathrm{T}_{\ddot{a}}(a_i) + \mathrm{I}_{\ddot{a}}(a_i) + \mathrm{F}_{\ddot{a}}(a_i) \le 3$, for all $a_i \in \ddot{Y}$.

(ii) The functions $\mathrm{T}_{b} : \varepsilon \subseteq \ddot{Y} \times \ddot{Y} \to [0,1]$, $\mathrm{I}_{b} : \varepsilon \subseteq \ddot{Y} \times \ddot{Y} \to [0,1]$ and $\mathrm{F}_{b} : \varepsilon \subseteq \ddot{Y} \times \ddot{Y} \to [0,1]$ denote the available (1), unsure (2), and unavailable (3) memberships of the edge (a_i, a_j) respectively, such that

$\mathrm{T}_{b}(a_i, a_j) \le \mathrm{low}[\mathrm{T}_{\ddot{a}}(a_i), \mathrm{T}_{\ddot{a}}(a_j)]$, ……………….. (1)

$\mathrm{I}_{b}(a_i, a_j) \le \mathrm{low}[\mathrm{I}_{\ddot{a}}(a_i), \mathrm{I}_{\ddot{a}}(a_j)]$, ………………... (2)

$\mathrm{F}_{b}(a_i, a_j) \le \mathrm{high}[\mathrm{F}_{\ddot{a}}(a_i), \mathrm{F}_{\ddot{a}}(a_j)]$……………….... (3)

and $0 \le \mathrm{T}_{b}(a_i, a_j) + \mathrm{I}_{b}(a_i, a_j) + \mathrm{F}_{b}(a_i, a_j) \le 3$,
for every edge (a_i, a_j).

Definition 2.3. [14]

A PNeuGr is stated in the form $G = (\ddot{Y}, \alpha, \beta)$, where the following conditions hold:

(i) Let $\mathrm{T}_{\ddot{a}} : \ddot{Y} \to [0,1]$, $\mathrm{I}_{\ddot{a}} : \ddot{Y} \to [0,1]$ and $\mathrm{F}_{\ddot{a}} : \ddot{Y} \to [0,1]$ denote the available, unsure, and unavailable memberships of the element $\ddot{u}_i \in \ddot{Y}$, respectively and $0 \le (\mathrm{T}_{\ddot{a}}(a_i))^2 + (\mathrm{I}_{\ddot{a}}(a_i))^2 + (\mathrm{F}_{\ddot{a}}(a_i))^2 \le 2$, for all $a_i \in \ddot{Y}$.

(ii) The functions $\mathrm{T}_{b} : \varepsilon \subseteq \ddot{Y} \times \ddot{Y} \to [0,1]$, $\mathrm{I}_{b} : \varepsilon \subseteq \ddot{Y} \times \ddot{Y} \to [0,1]$ and $\mathrm{F}_{b} : \varepsilon \subseteq \ddot{Y} \times \ddot{Y} \to [0,1]$ denote the available (4), unsure (5), and unavailable (6) memberships of the edge (a_i, a_j) respectively, such that

$\mathrm{T}_{b}(a_i, a_j) \le \mathrm{low}[\mathrm{T}_{\ddot{a}}(a_i), \mathrm{T}_{\ddot{a}}(a_j)]$, ……………. (4)

$\mathrm{I}_{b}(a_i, a_j) \le \mathrm{low}[\mathrm{I}_{\ddot{a}}(a_i), \mathrm{I}_{\ddot{a}}(a_j)]$, ……………… (5)

$\mathrm{F}_{b}(a_i, a_j) \le \mathrm{high}[\mathrm{F}_{\ddot{a}}(a_i), \mathrm{F}_{\ddot{a}}(a_j)]$ …………….. (6)

and $0 \le (\mathrm{T}_{b}(a_i))^2 + (\mathrm{I}_{b}(a_i))^2 + (\mathrm{F}_{b}(a_i))^2 \le 2$,
for every edge $(\ddot{u}_i, \ddot{u}_j)$.

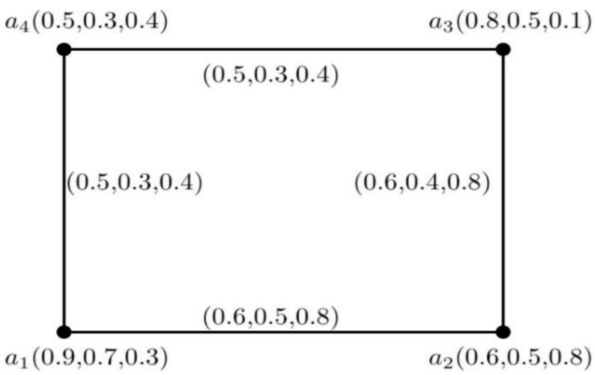

FIGURE 1. A Pythagorean Neutrosophic Graph.

Definition 2.4.

Consider $G = (\ddot{Y}, \alpha, \beta)$, as a PNeuGr. The degree $(\mathrm{T}, \mathrm{I}, \mathrm{F})$ of a vertex 'a' (7) is the summing of values of each membership of edges (8), (9), (10) that joins 'a', and it is denoted as $d_G(a)$.

(i.e.), $d_{G}(a) = (d_{Ţ}(a), d_{Ɨ}(a), d_{Ƒ}(a)$, ………… (7)
where
$d_{Ţ}(a) = \sum_{b \neq a} Ţ_{ƀ}(a, b)$, ……………………… (8)
$d_{Ɨ}(a) = \sum_{b \neq a} Ɨ_{ƀ}(a, b)$ ……………………….. (9)
and $d_{Ƒ}(a) = \sum_{b \neq a} Ƒ_{ƀ}(a, b)$………………….. (10)

Definition 2.5.

The min degree $(Ţ, Ɨ, Ƒ)$ of a PNeuGr $Ǥ = (Ÿ, α, β)$, is $δ(Ǥ) = (δ_{Ţ}(Ǥ), δ_{Ɨ}(Ǥ), δ_{Ƒ}(Ǥ))$, …………….. (11)
where
$δ_{Ţ}(Ǥ) = \min\{d_{Ţ}(b)/b ∈ B\}$, …………………. (12)
$δ_{Ɨ}(Ǥ) = \min\{d_{Ɨ}(b)/b ∈ B\}$ ………………….. (13)
and $δ_{Ƒ}(Ǥ) = \min\{d_{Ƒ}(b)/b ∈ B\}$ …………….. (14)

Equation (11) denote the min degree of a PNeuGr and (12), (13), (14) denotes the individual memberships of (11).

Definition 2.6.

The max degree $(Ţ, Ɨ, Ƒ)$ of a PNeuGr $Ǥ = (Ÿ, α, β)$ is $Δ(Ǥ) = (Δ_{Ţ}(Ǥ), Δ_{Ɨ}(Ǥ), Δ_{Ƒ}(Ǥ))$, ……………….. (15)
where
$Δ_{Ţ}(Ǥ) = \max\{d_{Ţ}(b)/b ∈ B\}$, ………………… (16)
$Δ_{Ɨ}(Ǥ) = \max\{d_{Ɨ}(b)/b ∈ B\}$ …………………. (17)
and $Δ_{Ƒ}(Ǥ) = \max\{d_{Ƒ}(b)/b ∈ B\}$…………….. (18)

Equation (15) denote the max degree of a PNeuGr and (16), (17), (18) denotes the individual memberships of (15).

III. Irregularity on Pythagorean Neutrosophic Graphs

Definition 3.1.

Consider a PNeuGr $Ǥ = (Ÿ, α, β)$. Then the vertex's neighborhood (NEI) is mentioned as,

NEI(a) = $(\mathrm{NEI}_{Ţ}(a), \mathrm{NEI}_{Ɨ}(a), \mathrm{NEI}_{Ƒ}(a))$, ……………… (19)
where
$\mathrm{NEI}_{Ţ}(a)$= {b∈ Ÿ: $Ţ_{ƀ}(a, b) \leq low[Ţ_{ä}(a), Ţ_{ä}(b)]$}, …. (20)
$\mathrm{NEI}_{Ɨ}(a)$ = {b ∈ Ÿ : $Ɨ_{ƀ}(a, b) \leq low[Ɨ_{ä}(a), Ɨ_{ä}(b)]$ }…….. (21)
$\mathrm{NEI}_{Ƒ}(a)$ = {b∈ Ÿ: $Ƒ_{ƀ}(a, b) \leq high[Ƒ_{ä}(a), Ƒ_{ä}(b)]$}…. (22)

Equation (19) denote the vertex's NEI and (20), (21), (22) denotes the individual memberships of (19).

Definition 3.2.

Consider a PNeuGr $Ǥ = (Ÿ, α, β)$. Then the vertex's NEI degree is mentioned as
deg(a) = $(deg_{Ţ}(a), deg_{Ɨ}(a), deg_{Ƒ}(a))$, ……….…... (23)
where
$deg_{Ţ}(a) = \sum_{b \in NEI_{Ţ}(a)} Ţ_{ä}(b)$, ……………………… (24)
$deg_{Ɨ}(a) = \sum_{b \in NEI_{Ɨ}(a)} Ɨ_{ä}(b)$, ……………………….. (25)
$deg_{Ƒ}(a) = \sum_{b \in NEI_{Ƒ}(a)} Ƒ_{ä}(b)$ ……………………… (26)

Equation (23) denote the vertex's NEI degree and (24), (25), (26) denotes the individual memberships of (23).

Definition 3.3.

Consider a PNeuGr $Ǥ = (Ÿ, α, β)$. Then the vertex's closed NEI degree is mentioned as
deg[a] = $(deg_{Ţ}[a], deg_{Ɨ}[a], deg_{Ƒ}[a])$, …………… (27)
where
$deg_{Ţ}$[a] = $\sum_{b \in NEI_{Ţ}(a)} Ţ_{ä}(b)$ + $Ţ_{ä}(a)$, ……………. (28)
$deg_{Ɨ}$[a] = $\sum_{b \in NEI_{Ţ}(a)} Ɨ_{ä}(b)$ + $Ɨ_{ä}(a)$, ………………. (29)
$deg_{Ƒ}$[a] = $\sum_{b \in NEI_{Ƒ}(a)} Ƒ_{ä}(b)$ + $Ƒ_{ä}(a)$ ……………… (30)

Equation (27) denote the vertex's closed NEI degree and (28), (29), (30) denotes the individual memberships of (27).

Definition 3.4.

Consider a PNeuGr $Ǥ = (Ÿ, α, β)$. Then the graph order Ord (Ǥ) is mentioned as
Ord (Ǥ) = $(Ord_{Ţ}(Ǥ), Ord_{Ɨ}(Ǥ), Ord_{Ƒ}(Ǥ))$, ……… (31)
where
$Ord_{Ţ}(Ǥ) = \sum_{a \in Ÿ} Ţ_{ä}(a)$, ………………………… (32)
$Ord_{Ɨ}(Ǥ) = \sum_{a \in Ÿ} Ɨ_{ä}(a)$, ………………………… (33)
$Ord_{Ƒ}(Ǥ) = \sum_{a \in Ÿ} Ƒ_{ä}(a)$………………………….. (34)

Equation (31) denote the order of PNeuGr and (32), (33), (34) denote the individual memberships of (31).

Definition 3.5.

Consider a PNeuGr $Ǥ = (Ÿ, α, β)$. Then the graph size Siz (Ǥ) is mentioned as
Siz (Ǥ) = $(Siz_{Ţ}(Ǥ), Siz_{Ɨ}(Ǥ), Siz_{Ƒ}(Ǥ))$, ……..….. (35)
where
$\mathrm{Siz}_{Ţ}(Ǥ) = \sum_{a \in Ÿ} Ţ_{ƀ}(a, b)$, ………………………. (36)
$\mathrm{Siz}_{Ɨ}(Ǥ) = \sum_{a \in Ÿ} Ɨ_{ƀ}(a, b)$, ………………………. (37)
$\mathrm{Siz}_{Ƒ}(Ǥ) = \sum_{a \in Ÿ} Ƒ_{ƀ}(a, b)$………………………… (38)

Equation (35) denote the size of PNeuGr and (36), (37), (38) denotes the individual memberships of (35).

Definition 3.6.

Let $Ǥ = (Ÿ, α, β)$ be a PNeuGr. Then the graph is called regular if all vertices have the equal NEI degree.

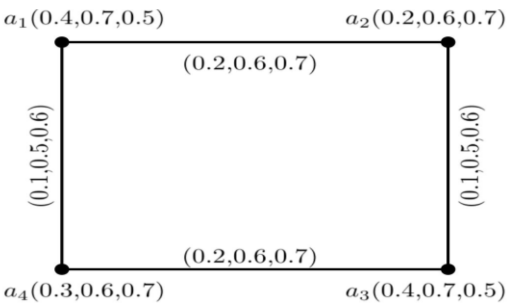

FIGURE 2. Regular Pythagorean Neutrosophic Graph.

Definition 3.7.

Let Ǥ = (Ÿ, α, β) be a PNeuGr. Then Ǥ is said to be an iPNeuGr, if there available a vertex that is adjacent to vertices with different NEI degrees.

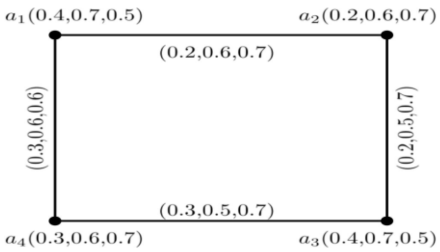

FIGURE 3. Irregular Pythagorean Neutrosophic Graph.

Definition 3.8.

Consider a PNeuGr Ǥ = (Ÿ, α, β). Then Ǥ is called a totally iPNeuGr if there available a vertex that is adjacent to vertices with different closed NEI degrees.

Definition 3.9.

Let Ǥ = (Ÿ, α, β) be a connected PNeuGr. Then Ǥ is called a NeiG iPNeuGr, if every vertex of a PNeuGr ends with distinct degrees. So, the two adjacent vertices of the graph also have distinct degree.

Definition 3.10.

Let Ǥ = (Ÿ, α, β) be a PNeuGr. Then Ǥ is called a NeiG totally iPNeuGr, if every vertex of a PNeuGr ends with different total degree. So, the same result will be obtained for every two adjacent vertices.

Definition 3.11.

Consider a connected PNeuGr Ǥ = (Ÿ, α, β). Then Ǥ is known to be HiG iPNeuGr, if all vertex of a PNeuGr ends with distinct degrees. So, the adjacent vertices for every vertex will have distinct degree.

Note 3.12.

1. A HiG irregular PNeuGr may not be a NeiG iPNeuGr.
2. A NeiG irregular PNeuGr may not be a HiG iPNeuGr.
3. A NeiG irregular PNeuGr may not be a NeiG totally iPNeuGr.
4. A NeiG totally irregular PNeuGr may not be a NeiG iPNeuGr.

Proposition 3.13.

A PNeuGr Ǥ = (Ÿ, α, β) is HiG irregular PNeuGr and NeiG iPNeuGr if and only if all the vertex degrees are distinct.

Proposition 3.14.

Consider a PNeuGr Ǥ = (Ÿ, α, β). If Ǥ = (Ÿ, α, β) is a NeiG iPNeuGr and M is a constant function then PNeuGr is a NeiG totally iPNeuGr.

Proposition 3.15.

Consider a PNeuGr Ǥ = (Ÿ, α, β). If Ǥ = (Ÿ, α, β) is a NeiG totally iPNeuGr and M denotes a constant function then PNeuGr is a NeiG iPNeuGr.

Proposition 3.16.

Consider a a PNeuGr Ǥ = (Ÿ, α, β). If PNeuGr is both NeiG iPNeuGr and NeiG totally iPNeuGr, then M need not be a constant function.

IV. Edge Irregularity on Pythagorean Neutrosophic Graphs

Definition 4.1.

Consider PNeuGr Ǥ = (Ÿ, α, β) to be a connected PNeuGr on $Ǥ_*$ = (Ÿ′, α′, β′). Then Ǥ is known to be:

(1) A NeiG edge iPNeuGr if all couple of adjacent edges have different degrees.

(2) A NeiG edge totally iPNeuGr if all couple of adjacent edges have different total degrees.

Theorem 4.2.

Consider a connected PNeuGr Ǥ = (Ÿ, α, β) on $Ǥ_*$ = (Ÿ′, α′, β′) and M: ($Ʈ_ƀ$, $Ɨ_ƀ$, $Ƒ_ƀ$) is a constant function. Then Ǥ is a NeiG edge iPNeuGr, iff Ǥ is a NeiG edge totally iPNeuGr.

Proof:

Let M: ($Ʈ_ƀ$, $Ɨ_ƀ$, $Ƒ_ƀ$) is a constant function, let β(ab) = K, ∀ab in edge set, where K = ($K_Ʈ$, $K_Ɨ$, $K_Ƒ$) is constant. Consider ab and bc as the couple of adjacent edges in edge set, then we have $d_Ǥ$(ab) ≠ $d_Ǥ$(bc),

↔ $d_Ǥ$(ab) + K ≠ $d_Ǥ$(bc) + K

↔ ($d_{Ʈ_ƀ}$(ab), $d_{Ɨ_ƀ}$(ab), $d_{Ƒ_ƀ}$(ab))+($K_Ʈ$, $K_Ɨ$, $K_Ƒ$) ≠ ($d_{Ʈ_ƀ}$(bc), $d_{Ɨ_ƀ}$(bc), $d_{Ƒ_ƀ}$(bc)) + ($K_Ʈ$, $K_Ɨ$, $K_Ƒ$)

↔ ($d_{Ʈ_ƀ}$(ab)+ $K_Ʈ$, $d_{Ɨ_ƀ}$(ab)+ $K_Ɨ$, $d_{Ƒ_ƀ}$(ab)+ $K_Ƒ$) ≠ ($d_{Ʈ_ƀ}$(bc)+$K_Ʈ$, $d_{Ɨ_ƀ}$(bc)+$K_Ɨ$, $d_{Ƒ_ƀ}$(bc)+$K_Ƒ$)

↔ ($d_{Ʈ_ƀ}$(ab)+$Ʈ_ƀ(ab)$, $d_{Ɨ_ƀ}$(ab)+$Ɨ_ƀ(ab)$, $d_{Ƒ_ƀ}$(ab)+$Ƒ_ƀ(ab)$) ≠ ($d_{Ʈ_ƀ}$(bc)+$Ʈ_ƀ(bc)$, $d_{Ɨ_ƀ}$(bc)+$Ɨ_ƀ(bc)$, $d_{Ƒ_ƀ}$(bc)+$Ƒ_ƀ(bc)$)

↔ ($td_{Ʈ_ƀ}$(ab), $td_{Ɨ_ƀ}$(ab), $td_{Ƒ_ƀ}$(ab))≠ ($td_{Ʈ_ƀ}$(bc), $td_{Ɨ_ƀ}$(bc), $td_{Ƒ_ƀ}$(bc))

↔ $td_Ǥ$(ab) ≠ $td_Ǥ$(bc) ………………………… (39)

By (39), all couple of adjacent edges have different degrees iff have different total degrees. This implies that, Ǥ is a NeiG edge iPNeuGr iff PNeuGr is a NeiG edge totally iPNeuGr.

Proposition 4.3.
Consider PNeuGr $Ǥ = (Ÿ, α, β)$ to be a connected PNeuGr on $Ǥ_* = (Ÿ', α', β')$. If Ǥ is both NeiG edge iPNeuGr and NeiG edge totally iPNeuGr, then M can't be necessarily a constant function.

Theorem 4.4.

Consider a connected PNeuGr $Ǥ = (Ÿ, α, β)$ on $Ǥ_* = (Ÿ', α', β')$ and M: $(Ƭ_ƀ, Ɨ_ƀ, Ƒ_ƀ)$ is a constant function. If Ǥ is a Str iPNeuGr, then Ǥ is a NeiG edge iPNeuGr.

Proof:

Consider a connected PNeuGr $Ǥ = (Ÿ, α, β)$ on $Ǥ_* = (Ÿ', α', β')$. Let M: $(Ƭ_ƀ, Ɨ_ƀ, Ƒ_ƀ)$ is a constant function, let β(ab) = K, ∀ab in edge set, where K = $(K_Ƭ, K_Ɨ, K_Ƒ)$ is constant. Consider ab and bc as the couple of adjacent edges in edge set. Suppose that Ǥ is a Str iPNeuGr. Then all couple of vertices in Ǥ have distinct degrees, which results in,

$d_Ǥ(a) \neq d_Ǥ(b) \neq d_Ǥ(c)$

$\rightarrow (d_{Ƭ_ä}(a), d_{Ɨ_ä}(a), d_{Ƒ_ä}(a)) \neq (d_{Ƭ_ä}(b), d_{Ɨ_ä}(b), d_{Ƒ_ä}(b)) \neq (d_{Ƭ_ä}(c), d_{Ɨ_ä}(c), d_{Ƒ_ä}(c))$

$\rightarrow (d_{Ƭ_ä}(a), d_{Ɨ_ä}(a), d_{Ƒ_ä}(a)) + (d_{Ƭ_ä}(b), d_{Ɨ_ä}(b), d_{Ƒ_ä}(b)) - 2(K_Ƭ, K_Ɨ, K_Ƒ) \neq$
$(d_{Ƭ_ä}(b), d_{Ɨ_ä}(b), d_{Ƒ_ä}(b)) + (d_{Ƭ_ä}(c), d_{Ɨ_ä}(c), d_{Ƒ_ä}(c)) - 2(K_Ƭ, K_Ɨ, K_Ƒ)$

$\rightarrow (d_{Ƭ_ƀ}(ab), d_{Ɨ_ƀ}(ab), d_{Ƒ_ƀ}(ab)) \neq$
$(d_{Ƭ_ƀ}(bc), d_{Ɨ_ƀ}(bc), d_{Ƒ_ƀ}(bc))$

$\rightarrow d_Ǥ(ab) \neq d_Ǥ(bc)$………………………… (40)

By (40), all couple of adjacent edges have different degrees, therefore Ǥ is a NeiG edge iPNeuGr.

Theorem 4.5.

Consider a connected PNeuGr $Ǥ = (Ÿ, α, β)$ on $Ǥ_* = (Ÿ', α', β')$ and M: $(Ƭ_ƀ, Ɨ_ƀ, Ƒ_ƀ)$ is a constant function. If Ǥ is a HiG iPNeuGr, then Ǥ is a NeiG edge iPNeuGr.

Proof:

Consider a connected PNeuGr $Ǥ = (Ÿ, α, β)$ on $Ǥ_* = (Ÿ', α', β')$. Let M: $(Ƭ_ƀ, Ɨ_ƀ, Ƒ_ƀ)$ is a constant function, let β(ab) = K, ∀ab in edge set, where K = $(K_Ƭ, K_Ɨ, K_Ƒ)$ is constant. Consider ab and bc as the couple of adjacent edges in edge set. Suppose that Ǥ is a Str iPNeuGr. Then all couple of vertices in Ǥ have different degrees, which results in, $d_Ǥ(a) \neq d_Ǥ(c)$

$\rightarrow (d_{Ƭ_ä}(a), d_{Ɨ_ä}(a), d_{Ƒ_ä}(a)) \neq (d_{Ƭ_ä}(c), d_{Ɨ_ä}(c), d_{Ƒ_ä}(c))$

$\rightarrow (d_{Ƭ_ä}(a), d_{Ɨ_ä}(a), d_{Ƒ_ä}(a)) + (d_{Ƭ_ä}(b), d_{Ɨ_ä}(b), d_{Ƒ_ä}(b)) - 2(K_Ƭ, K_Ɨ, K_Ƒ) \neq$
$(d_{Ƭ_ä}(b), d_{Ɨ_ä}(b), d_{Ƒ_ä}(b)) + (d_{Ƭ_ä}(c), d_{Ɨ_ä}(c), d_{Ƒ_ä}(c)) - 2(K_Ƭ, K_Ɨ, K_Ƒ)$

$\rightarrow (d_{Ƭ_ƀ}(ab), d_{Ɨ_ƀ}(ab), d_{Ƒ_ƀ}(ab)) \neq (d_{Ƭ_ƀ}(bc), d_{Ɨ_ƀ}(bc), d_{Ƒ_ƀ}(bc))$

$\rightarrow d_Ǥ(ab) \neq d_Ǥ(bc)$………………………… (41)

By (41), all couple of adjacent edges have different degrees, iff all vertex adjacent to the vertices have different degrees. Therefore, Ǥ is a HiG iPNeuGr iff Ǥ is a NeiG edge iPNeuGr.

Definition 4.6.

Consider PNeuGr $Ǥ = (Ÿ, α, β)$ to be a connected PNeuGr on $Ǥ_* = (Ÿ', α', β')$. Then Ǥ is known to be:

(1) A Str edge iPNeuGr if all couple of edges have different degrees.

(2) A Str edge totally iPNeuGr if all couple of edges have different total degrees.

Theorem 4.7.

Consider a connected PNeuGr $Ǥ = (Ÿ, α, β)$ on $Ǥ_* = (Ÿ', α', β')$ and M: $(Ƭ_ƀ, Ɨ_ƀ, Ƒ_ƀ)$ is a constant function. Then Ǥ is a Str edge iPNeuGr, iff Ǥ is a Str edge totally iPNeuGr.

Proof:

Let M: $(Ƭ_ƀ, Ɨ_ƀ, Ƒ_ƀ)$ is a constant function, let β(ab) = K, ∀ab in edge set, where K = $(K_Ƭ, K_Ɨ, K_Ƒ)$ is constant. Consider ab and bc as the couple of adjacent edges in edge set, then we have $d_Ǥ(ab) \neq d_Ǥ(cd)$,

$\leftrightarrow d_Ǥ(ab) + K \neq d_Ǥ(cd) + K$

$\leftrightarrow (d_{Ƭ_ƀ}(ab), d_{Ɨ_ƀ}(ab), d_{Ƒ_ƀ}(ab)) + (K_Ƭ, K_Ɨ, K_Ƒ) \neq (d_{Ƭ_ƀ}(cd), d_{Ɨ_ƀ}(cd), d_{Ƒ_ƀ}(cd)) + (K_Ƭ, K_Ɨ, K_Ƒ)$

$\leftrightarrow (d_{Ƭ_ƀ}(ab) + K_Ƭ, d_{Ɨ_ƀ}(ab) + K_Ɨ, d_{Ƒ_ƀ}(ab) + K_Ƒ) \neq (d_{Ƭ_ƀ}(cd) + K_Ƭ, d_{Ɨ_ƀ}(cd) + K_Ɨ, d_{Ƒ_ƀ}(cd) + K_Ƒ)$

$\leftrightarrow (d_{Ƭ_ƀ}(ab) + Ƭ_ƀ(ab), d_{Ɨ_ƀ}(ab) + Ɨ_ƀ(ab), d_{Ƒ_ƀ}(ab) + Ƒ_ƀ(ab)) \neq (d_{Ƭ_ƀ}(cd) + Ƭ_ƀ(cd), d_{Ɨ_ƀ}(cd) + Ɨ_ƀ(cd), d_{Ƒ_ƀ}(cd) + Ƒ_ƀ(cd))$

$\leftrightarrow (td_{Ƭ_ƀ}(ab), td_{Ɨ_ƀ}(ab), td_{Ƒ_ƀ}(ab)) \neq$
$(td_{Ƭ_ƀ}(cd), td_{Ɨ_ƀ}(cd), td_{Ƒ_ƀ}(cd))$

$\leftrightarrow td_Ǥ(ab) \neq td_Ǥ(cd)$ ……………………… (42)

Therefore by (42), all couple of adjacent edges have different degrees iff have different total degrees. This implies that, Ǥ is a Str edge iPNeuGr iff PNeuGr is a Str edge totally iPNeuGr.

Proposition 4.8.
Consider PNeuGr $Ǥ = (Ϋ, α, β)$ to be a connected PNeuGr on $Ǥ_* = (Ϋ', α', β')$. If Ǥ is both Str edge iPNeuGr and Str edge totally iPNeuGr, then M can't be necessarily a constant function.

Theorem 4.9.

Consider a connected PNeuGr $Ǥ = (Ϋ, α, β)$ on $Ǥ_* = (Ϋ', α', β')$ and M: ($Ʈ_{ƀ}$,$Ɨ_{ƀ}$,$Ƒ_{ƀ}$) is a constant function. If Ǥ is a Str edge iPNeuGr, then Ǥ is an iPNeuGr.

Proof:

Consider a connected PNeuGr $Ǥ = (Ϋ, α, β)$ on $Ǥ_* = (Ϋ', α', β')$. Let M: ($Ʈ_{ƀ}$,$Ɨ_{ƀ}$,$Ƒ_{ƀ}$) is a constant function, let β(ab) = K, ∀ab in edge set, where K = ($K_{Ʈ}$, $K_{Ɨ}$,$K_{Ƒ}$) is constant. Consider ab and bc as the couple of adjacent edges in edge set. Suppose that Ǥ is a Str iPNeuGr. Then all couple of vertices in Ǥ have different degrees, which results in, $d_{Ǥ}$(ab) ≠ $d_{Ǥ}$(bc)

→ ($d_{Ʈ_{ƀ}}$(ab),$d_{Ɨ_{ƀ}}$(ab),$d_{Ƒ_{ƀ}}$(ab)) ≠ ($d_{Ʈ_{ƀ}}$(bc),$d_{Ɨ_{ƀ}}$(bc),$d_{Ƒ_{ƀ}}$(bc))

→ ($d_{Ʈ_{ä}}$(a) + $d_{Ʈ_{ä}}$(b) - 2$Ʈ_{ƀ}$(ab), $d_{Ɨ_{ä}}$(a) + $d_{Ɨ_{ä}}$(b) - 2$Ɨ_{ƀ}$(ab), $d_{Ƒ_{ä}}$(a) + $d_{Ƒ_{ä}}$(b) - 2$Ƒ_{ƀ}$(ab)) ≠
($d_{Ʈ_{ä}}$(b) + $d_{Ʈ_{ä}}$(c) - 2$Ʈ_{ƀ}$(bc), $d_{Ɨ_{ä}}$(b) + $d_{Ɨ_{ä}}$(c) - 2$Ɨ_{ƀ}$(bc), $d_{Ƒ_{ä}}$(b) + $d_{Ƒ_{ä}}$(c) - 2$Ƒ_{ƀ}$(bc))

→ $d_{Ǥ}$(a) + $d_{Ǥ}$(b) ≠ $d_{Ǥ}$(b) + $d_{Ǥ}$(c)

→ $d_{Ǥ}$(a) ≠ $d_{Ǥ}$(c) ………………….……… (43)

By (43), there is a vertex b, which is adjacent to vertices a and c have different degrees. Therefore, Ǥ is an iPNeuGr.

Theorem 4.10.

Consider a connected PNeuGr $Ǥ = (Ϋ, α, β)$ on $Ǥ_* = (Ϋ', α', β')$ and M: ($Ʈ_{ƀ}$,$Ɨ_{ƀ}$,$Ƒ_{ƀ}$) is a constant function. If Ǥ is a Str edge iPNeuGr, then Ǥ is a HiG iPNeuGr.

Proof:

Consider a connected PNeuGr $Ǥ = (Ϋ, α, β)$ on $Ǥ_* = (Ϋ', α', β')$. Let M: ($Ʈ_{ƀ}$,$Ɨ_{ƀ}$,$Ƒ_{ƀ}$) is a constant function, let β(ab) = K, ∀ab in edge set, where K = ($K_{Ʈ}$, $K_{Ɨ}$,$K_{Ƒ}$) is constant. Consider ab and bc as the couple of adjacent edges in edge set. Suppose that Ǥ is a Str iPNeuGr. Then all couple of vertices in Ǥ have different degrees, which results in, $d_{Ǥ}$(ab) ≠ $d_{Ǥ}$(bc) ≠ $d_{Ǥ}$(bd)

→ ($d_{Ʈ_{ƀ}}$(ab),$d_{Ɨ_{ƀ}}$(ab),$d_{Ƒ_{ƀ}}$(ab)) ≠ ($d_{Ʈ_{ƀ}}$(bc),$d_{Ɨ_{ƀ}}$(bc),$d_{Ƒ_{ƀ}}$(bc)) ≠ ($d_{Ʈ_{ƀ}}$(bd),$d_{Ɨ_{ƀ}}$(bd),$d_{Ƒ_{ƀ}}$(bd))

→ ($d_{Ʈ_{ä}}$(a) + $d_{Ʈ_{ä}}$(b) - 2$Ʈ_{ƀ}$(ab), $d_{Ɨ_{ä}}$(a) + $d_{Ɨ_{ä}}$(b) - 2$Ɨ_{ƀ}$(ab), $d_{Ƒ_{ä}}$(a) + $d_{Ƒ_{ä}}$(b) - 2$Ƒ_{ƀ}$(ab)) ≠

($d_{Ʈ_{ä}}$(b) + $d_{Ʈ_{ä}}$(c) - 2$Ʈ_{ƀ}$(bc), $d_{Ɨ_{ä}}$(b) + $d_{Ɨ_{ä}}$(c) - 2$Ɨ_{ƀ}$(bc), $d_{Ƒ_{ä}}$(b) + $d_{Ƒ_{ä}}$(c) - 2$Ƒ_{ƀ}$(bc)) ≠

($d_{Ʈ_{ä}}$(b) + $d_{Ʈ_{ä}}$(d) - 2$Ʈ_{ƀ}$(bd), $d_{Ɨ_{ä}}$(b) + $d_{Ɨ_{ä}}$(d) - 2$Ɨ_{ƀ}$(bd), $d_{Ƒ_{ä}}$(b) + $d_{Ƒ_{ä}}$(d) - 2$Ƒ_{ƀ}$(bd))

→ $d_{Ǥ}$(a) + $d_{Ǥ}$(b) ≠ $d_{Ǥ}$(b) + $d_{Ǥ}$(c) ≠ $d_{Ǥ}$(b) + $d_{Ǥ}$(d)

→ $d_{Ǥ}$(a) ≠ $d_{Ǥ}$(c) ≠ $d_{Ǥ}$(d)…………………. (44)

By (44), the vertex b is adjacent to the vertices with different degrees. Therefore, Ǥ is a HiG iPNeuGr.

V. APPLICATION

A PNeuGr is an advanced structure formed based on Pythagorean NeuS with graphs. It is used to model uncertain and inconsistent ideas in systems of numerous fields:

(a) Decision Making:
When hiring an employee, the manager or HR faces a critical problem in dealing with the conflicting choices involving inconsistency. In this case, the vertices represent the alternatives and the edges represent the pairwise comparisons. This approach incorporates hesitation that makes the decision stronger under uncertainty.

(b) Social Network:
The uncertain relationships in modeling social media or communities are taken with the context of human relationships. Here, the individuals are assumed as vertices, and edge membership is allotted as trust, neutrality, and distrust. This benefits by analyzing the communities where connections are made with ambiguous behavior.

(c) Risk Assessment:
The cyber system is used to evaluate the uncertain threats in networks. The vertices are taken as server or database components, and the edges capture the risk likelihood, doubt, and improbability as membership. This model is more realistic for analyzing the risk in complex environments.

(d) Transportation:
Planning routes based on traffic and weather conditions is modeled using the neutrosophic vertex and edge membership values. Locations can be taken as vertices, and the edges are assumed to be the paths with unfixed travel times. This structure optimizes the plan routes for average time, reliability, and uncertainty.

Practical role of irregularities on PNeuGr:
A heterogeneous structure involves high irregularity, critical in designing a network and assessing threats.

The vertices with high irregularity are considered weak points in cybersecurity. The clusters in PNeuGr show low internal and high external irregularities, which helps segment recommendation systems. The profile of irregularity is implemented by comparing the different patient-symptom graphs for similar diseases.

VI. CONCLUSION

This manuscript encloses a detail discussion on some insights like irregular and edge irregular properties on PNeuGr. Various kinds are irregularities and edge irregularities are compared and attained through theorem results. In future, we planned to execute the PNeuGr with some other graphical schemes and properties.

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Govindan Vetrivel: Conceptualization, Data Curation, Methodology, Validation, Writing – Original Draft Preparation;

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CONFLICT OF INTERESTS

No conflict of interests was disclosed.

ETHICS STATEMENTS

This research work did not involve human participants, animal subjects, or sensitive personal data, and therefore did not require ethical approval.

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