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MATHEMATICAL MODELING ON THE TRANSMISSION DYNAMICS OF ZIKA VIRUS

Murugappan Mullai*, G. Madhan Kumar, Grienggrai Rajchakit* and Govindan Vetrivel

***Abstract* – Zika virus is a mosquito-borne virus that is commonly transmitted by mosquitoes of the Aedes genus. The transmission dynamics of the Zika virus in males, females, and children are comparatively studied. The study aims to analyze and find the population that affects more due to Zika transmission. This paper deals with the non-linear Mathematical model of the dynamics of Zika virus transmission. The reproductive ratio of the model is calculated to analyze the spread of the Zika virus. The equilibrium and the stability of the model are found and analyzed analytically. Numerical simulation is carried out to support the analytical results and to estimate the most affecting population in different equilibria.**

Keywords—Mathematical model, Zika virus, Equilibrium, Stability.

1. INTRODUCTION

The aim of this paper deals with the non-linear mathematical model of the Zika virus and its spread in the human population. The Zika virus is a flavivirus dispersed by mosquitos that originated in monkeys in Uganda in 1947 as part of a yellow fever surveillance network. It was later observed in civilians of Uganda and Tanzania's United Republic in 1952[3]. Infections with the Zika virus have been documented in Asia, the Pacific, the Americas, and Africa. Yap Island in Micronesia saw a Zika virus outbreak in 2007 that was caused by the Asian family of the virus (ZIKV) [4, 5]. The recent emergence of Zika demonstrates that diseases that spread primarily through other means can also have a sexual component to their spread [6, 7]. A pregnant woman who contracts the mosquito-borne Zika virus runs the risk of giving birth to a child with birth abnormalities [8,9]. The Zika virus is typically transmitted to humans by the bite of an infected mosquito of the Aedes genus, most notably Aedes aegypti in tropical areas. Aedes mosquitos bite most often throughout the day, peaking in the early morning and late afternoon/evening. This is the mosquito that spreads dengue disease, chikungunya, and yellow fever. The Zika virus may also spread via sexual activity. Blood transfusion and other mechanisms of transmission are being researched. F.B.Agusto et al.[1] framed a mathematical model of the Zika virus and discussed its vertical transmission, and also Joel C.Miller[12] discussed mathematical models of SIR disease and sexual transmission routes. P. Suparit et al. [15] proceed with a time-dependent mosquito bite to model the Zika spread. M. Rahman et al. [9] studied and recorded the parameters, disease characteristics, and prevention of Zika transmission. Using fractional order derivative, Shahram Rezapour et al. [17] furnished a math model for Zika transmission. In 2020, S.K. Biswas et al. [18] formed a mathematical model to study the

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Zika virus dynamics by considering the population of both humans and mosquitoes. A case study in India has been conducted on Zika virus progression dynamics using the SIR-SI math model by Ravins Dohare et al. [16]. By considering the nonlinear general incidence rate, Ahmed Alshehri and Miled El hajji [2] undergoes a study on Zika transmission in 2022. Recently, a mathematical study to learn about the infection of the Zika virus with microcephaly threat was conducted and presented by Mahmoud A. Ibrahim and Attila Denes [6]. S.K. Biswas et al. [19] submitted a case study on Zika transmission with saturated incidence and optimal control.

In this paper, a mathematical model of zika transmission was formulated and studied with the population of males, females and children. The previous and recent studies don't have a comparative study among the human and gender populations. Here we considered three compartments viz., Susceptible, Infected, and Recovered to study the transmission dynamics of Zika virus. The equilibrium and its stability results are analytically obtained and numerical simulation, which portrays a clear picture of the most affected population by Zika. The analytical and numerical study results of Zika transmission for a certain period (year-wise) can be recorded and it provides clear ups and downs of Zika spread with the altering increasing and decreasing rate, seasonal spread scale, etc. The significance of the study is quite useful to take initial precautionary measures to control the virus spread by clearing the stagnant water, eradicating it through mosquito spray and pesticides, creating awareness among the public, etc. The government and private agencies can be alerted at the right time by using this study results.

2. Model Formulation

Consider, N = $S_m + I_m + S_f + I_f + S_c + I_c$ + R

where,

- N denotes the total population.
- S_m , S_f and S_c denote the number of Susceptible males, Susceptible females and Susceptible children population who may or may not get infected with Zika virus.
- I_m , I_f , I_c denote the number of Infected males, Infected females and Infected children population who got infected with Zika virus
- R denotes the number of recovered individuals (male, female and child).

The mathematical model is proposed as follows: -

$$\frac{dS_m}{dt} = \Lambda_1 - \alpha_1(1+\theta)\, S_m I_m - \alpha_2 S_m I_f - \mu_1 S_m$$

$$\frac{dI_m}{dt} = \alpha_1(1+\theta)\, S_m I_m + \alpha_2 S_m I_f - \tau_1 I_m - \mu_1 I_m$$

$$\frac{dS_f}{dt} = \Lambda_2 - \beta_1(1+\theta)\, S_f I_f - \beta_2 S_f I_m - \mu_2 S_f$$

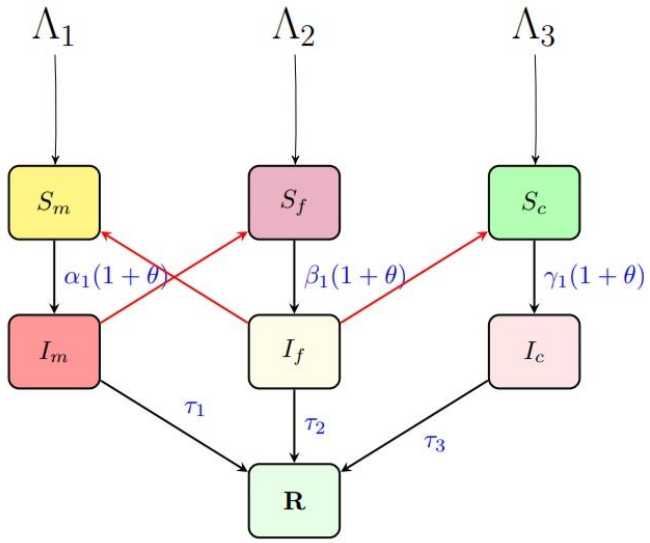
(2.1)
$$\frac{dI_f}{dt} = \beta_1(1+\theta)\, S_f I_f + \beta_2 S_f I_m - \tau_2 I_f - \mu_2 I_f$$

$$\frac{dS_c}{dt} = \Lambda_3 - \gamma_1(1+\theta)\, S_c I_c - \gamma_2 S_c I_f - \mu_3 S_c$$

$$\frac{dI_c}{dt} = \gamma_1(1+\theta)\, S_c I_c + \gamma_2 S_c I_f - \tau_3 I_c - \mu_3 I_c$$

$$\frac{dR}{dt} = \tau_1 I_m + \tau_2 I_f + \tau_3 I_c - \mu R$$

The transfer diagram of our model is shown below:



The parameters used in the model are described below

Parameters	Description
Λ_1	Male recruitment rate
Λ_2	Female recruitment rate
Λ_3	Child recruitment rate
μ_1	Male natural death rate
μ_2	Female natural death rate
μ_3	Child natural death rate
$\alpha_1(1+\theta)$, $\beta_1(1+\theta)$ and $\gamma_1(1+\theta)$	Rate of transmission from S to I
α_2, β_2 and γ_2	Rate of transmission from S to I
τ_1, τ_2 and τ_3	Treatment Rate

3. Male and Female Population

$$\frac{dS_m}{dt} = \Lambda_1 - \alpha_1(1+\theta)\, S_m I_m - \alpha_2 S_m I_f - \mu_1 S_m$$

$$\frac{dI_m}{dt} = \alpha_1(1+\theta)\, S_m I_m + \alpha_2 S_m I_f - \tau_1 I_m - \mu_1 I_m$$

$$\frac{dS_f}{dt} = \Lambda_2 - \beta_1(1+\theta) S_f I_f - \beta_2 S_f I_m - \mu_2 S_f$$

$$(2.2)\quad \frac{dI_f}{dt} = \beta_1(1+\theta) S_f I_f + \beta_2 S_f I_m - \tau_2 I_f - \mu_2 I_f$$

$$\frac{dR}{dt} = \tau_1 I_m + \tau_2 I_f - \mu R$$

Case 1: I_m=0

Now, the model exhibits two equilibria, namely,

(2.3) $I_0 = (S_m^0, S_f^0, I_f^0, R^0) = (\frac{\lambda_1}{\mu_1}, \frac{\lambda_2}{\mu_2}, 0, 0)$

(2.4) $I^* = (S_m^*, S_f^*, I_f^*, R^*)$

where,

$$S_m^* = \frac{\Lambda_1}{\alpha_2 I_f^* + \mu_1}$$

$$S_f^* = \frac{\tau_2 + \mu_2}{\beta_1(1+\theta)}$$

$$I_f^* = \frac{\lambda_2 - \mu_2 S_f^*}{\beta_1(1+\theta) S_f^*}$$

$$R^* = \frac{\tau_2 I_f^*}{\mu}$$

Theorem 2.1. The disease-free equilibrium point exists only when $R_0 < 1$.

Proof. The evaluated jacobian at disease free point is given by

(2.5) J=
$$\begin{pmatrix} -\mu_1 & 0 & -\alpha_2 \frac{\Lambda_1}{\mu_1} & 0 \\ 0 & -\mu_2 & -\beta_1(1+\theta)\frac{\Lambda_1}{\mu_1} & 0 \\ 0 & 0 & \beta_1(1+\theta)\frac{\Lambda_1}{\mu_1} - \tau_2 - \mu_2 & 0 \\ 0 & 0 & \tau_2 & -\mu \end{pmatrix}$$

After linearization,

(2.6)

$$\begin{pmatrix} -\mu_1 - \lambda & 0 & -\alpha_2 \frac{\Lambda_1}{\mu_1} & 0 \\ 0 & -\mu_2 - \lambda & -\beta_1(1+\theta)\frac{\Lambda_1}{\mu_1} & 0 \\ 0 & 0 & \beta_1(1+\theta)\frac{\Lambda_1}{\mu_1} - \tau_2 - \mu_2 - \lambda & 0 \\ 0 & 0 & \tau_2 & -\mu - \lambda \end{pmatrix}$$

which gives the characteristic equation

$(-\mu_1 - \lambda)(-\mu_2 - \lambda)(\beta_1(1+\theta)\frac{\Lambda_1}{\mu_1} - \tau_2 - \mu_2 - \lambda)(-\mu - \lambda) = 0$

where,

$$\beta_1(1+\theta)\frac{\Lambda_1}{\mu_1} - \tau_2 - \mu_2 - \lambda = 0$$

$$\lambda = \beta_1(1+\theta)\frac{\Lambda_1}{\mu_1} - \tau_2 - \mu_2 < 0$$

$$\beta_1(1+\theta)\frac{\Lambda_1}{\mu_1} < (\tau_2 + \mu_2)$$

$$\frac{\beta_1(1+\theta)\frac{\Lambda_1}{\mu_1}}{\tau_2 + \mu_2} < 1$$

i.e., $R_0 < 1$.

Therefore, the disease-free equilibrium point exists only when $R_0 < 1$.

Case 2: $I_f = 0$

Now, the model exhibits two equilibria namely,

(2.7) $E_0 = (S_m^0, I_f^0, S_f^0, R^0) = (\frac{\lambda_1}{\mu_1}, 0, \frac{\lambda_2}{\mu_2}, 0)$

(2.8) $E^* = (S_m^*, I_m^*, S_f^*, R^*)$

where,

$$S_m^* = \frac{\tau_1 + \mu_1}{\alpha_1(1+\theta)}$$

$$I_m^* = \frac{\Lambda_1 - \mu_1 S_m^*}{\alpha_1(1+\theta) S_m^*}$$

$$S_f^* = \frac{\lambda_2}{\beta_2 I_m^* + \mu_2}$$

$$R^* = \frac{\tau_1 I_m^*}{\mu}$$

4. Female and Child Population

$$\frac{dS_f}{dt} = \Lambda_2 - \beta_1(1+\theta) S_f I_f - \beta_2 S_f I_m - \mu_2 S_f$$

$$(2.9)\quad \frac{dI_f}{dt} = \beta_1(1+\theta) S_f I_f + \beta_2 S_f I_m - \tau_2 I_f - \mu_2 I_f$$

$$\frac{dS_c}{dt} = \Lambda_3 - \gamma_1(1+\theta) S_c I_c - \gamma_2 S_c I_f - \mu_3 S_c$$

$$\frac{dI_c}{dt} = \gamma_1(1+\theta) S_c I_c + \gamma_2 S_c I_f - \tau_3 I_c - \mu_3 I_c$$

$$\frac{dR}{dt} = \tau_2 I_f + \tau_3 I_c - \mu R$$

Case 1: $I_f = 0$

Now, the model exhibits two equilibria, namely,

(2.10) $F_0 = (S_f^0, I_c^0, S_c^0, R^0) = (\frac{\Lambda_2}{\mu_2}, \frac{\Lambda_3}{\mu_3}, 0, 0)$

(2.11) $F^* = (S_f^*, I_c^*, S_c^*, R^*)$

where,

$$S_f^* = \frac{\lambda_2}{\mu_2}$$

$$S_c^* = \frac{\tau_3 + \mu_3}{\gamma_1(1+\theta)}$$

$$I_c^* = \frac{\Lambda_3 - \mu_3 S_c^*}{\gamma_1(1+\theta)S_c^*}$$

$$R^* = \frac{\tau_3\, I_c^*}{\mu}$$

Theorem 2.2. The disease-free equilibrium point exists only when $R_0 < 1$.

Proof. The evaluated jacobian at disease free point is given by

$$\text{(2.12)} \qquad J=\begin{pmatrix} -\mu_2 & 0 & 0 & 0 \\ 0 & -\mu_3 & 0 & 0 \\ 0 & 0 & \gamma_1(1+\theta)\frac{\Lambda_3}{\mu_3} - \tau_3 - \mu_3 & 0 \\ 0 & 0 & \tau_3 & -\mu \end{pmatrix}$$

After linearization,

$$\text{(2.13)} \begin{pmatrix} -\mu_2 - \lambda & 0 & 0 & 0 \\ 0 & -\mu_3 - \lambda & 0 & 0 \\ 0 & 0 & \gamma_1(1+\theta)\frac{\Lambda_3}{\mu_3} - \tau_3 - \mu_3 - \lambda & 0 \\ 0 & 0 & \tau_3 & -\mu - \lambda \end{pmatrix}$$

which gives the characteristic equation

$(-\mu_2 - \lambda)(-\mu_3 - \lambda)(\gamma_1(1+\theta)\frac{\Lambda_3}{\mu_3} - \tau_3 - \mu_3 - \lambda)(-\mu - \lambda)$ =0

where,

$$\gamma_1(1+\theta)\frac{\Lambda_3}{\mu_3} - \tau_3 - \mu_3 - \lambda = 0$$

$$\lambda = \gamma_1(1+\theta)\frac{\Lambda_3}{\mu_3} - \tau_3 - \mu_3 < 0$$

$$\gamma_1(1+\theta)\frac{\Lambda_3}{\mu_3} < (\tau_3 + \mu_3)$$

$$\frac{\gamma_1(1+\theta)\frac{\Lambda_3}{\mu_3}}{\tau_3 + \mu_3} < 1$$

i.e., $R_0 < 1$.

Therefore, the disease-free equilibrium point exists only when $R_0 < 1$.

Case 1: $I_c = 0$

Now, the model exhibits two equilibria, namely,

$$\text{(2.14)} \qquad G_0 = \left(S_f^0, I_f^0, S_c^0, R^0\right) = \left(\frac{\Lambda_2}{\mu_2}, 0, \frac{\Lambda_3}{\mu_3}, 0\right)$$

$$\text{(2.15)} \qquad G^* = \left(S_f^*, I_f^*, S_c^*, R^*\right)$$

where,

$$S_f^* = \frac{\tau_2 + \mu_2}{\beta_1(1+\theta)}$$

$$I_f^* = \frac{\Lambda_2 - \mu_2 S_f^*}{\tau_2 + \mu_2}$$

$$S_c^* = \frac{\Lambda_3}{\gamma_2 I_f^* + \mu_3}$$

$$R^* = \frac{\tau_2\, I_f^*}{\mu}$$

5. Numerical Simulation

The numerical simulation signifies the relationship between time and population. The model is discussed under two criteria: 1) For Males and Females, 2) For Female and Children. This study helps us to find the population that is highly and lowly affected due to zika spread. For a total of 500 days, the ups and downs of the population can be noted for every interval gap of 50 days. The final results show that the male and female population is highly affected when compared to other population in I_0 equilibrium (Fig 1 & 4), where the basic reproduction number (R_0) is less than 1. In I^* equilibrium, the infected and recovered rate of the population is high, where the basic reproduction number (R_0) is greater than 1 (Fig 2 & 3).

A Numerical simulation that relates the susceptible male (S_m), susceptible female (S_f), Infected (I), and Recovered (R), under I_0 and I^* equilibrium, is shown in Fig 1 & 2. A Simulation that relates susceptible females (S_f), susceptible children (S_c), Infected (I), and Recovered (R) under I^* and I_0 equilibrium is shown in Fig 3 & 4.

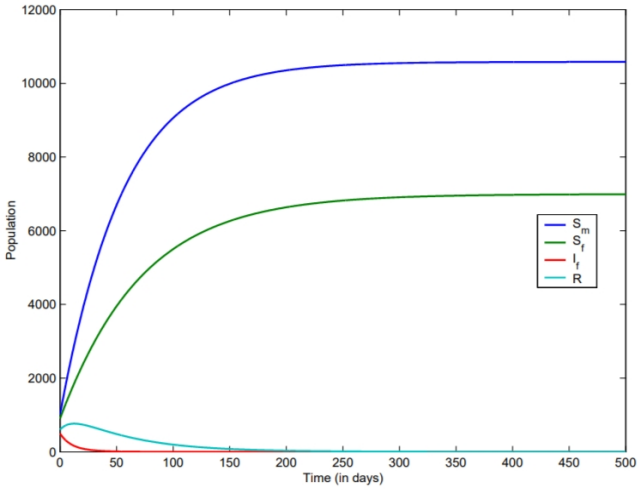


Figure 1: Existence of I_0 equilibrium point at R_0 = 0.6718 < 1

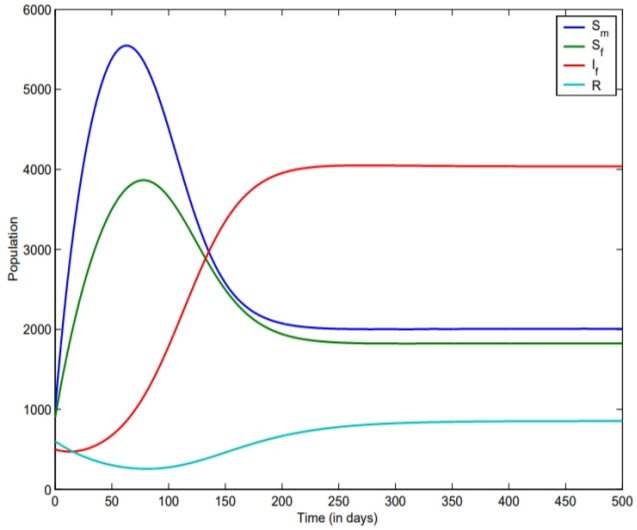


Figure 2: Existence of I^* equilibrium point at R_0 = 3.8290 > 1

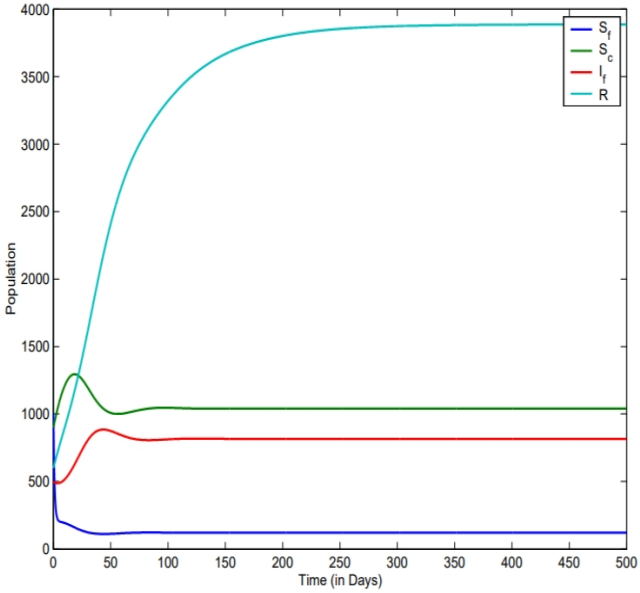


Figure 3: Existence of I^* equilibrium point at R_0 = 6.0718 > 1

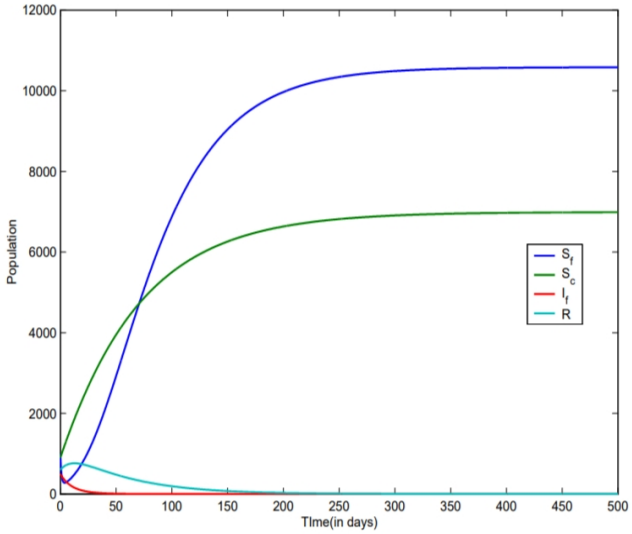


Figure 4: Existence of I_0 equilibrium point at R_0 = 0.6718 < 1

6. Conclusion

In this manuscript, we framed and analyzed a non-linear mathematical model regarding Zika virus transmission between male, female, and child populations. The equilibria of the model are found, and the reproduction ratio is determined, i.e., when R_0 < 1 disease-free state exists and R_0 > 1 endemic state exists. The stability of the model is shown graphically, and numerical simulation is carried out to support the analytical findings. Through the numerical simulations, we conclude that the male and female populations are highly affected when compared with other populations, at R_0 < 1. At R_0 > 1, the infected and recovered rate is higher than the taken joint populations. In the future, we plan to execute a math model with another disease transmission, economic crisis, etc.

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