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Viscous Dissipation Effects in A Microchannel Caused by Oscillation of One Surface

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***Abstract* – The viscous dissipation effects in a microchannels caused by an oscillatory lower surface is investigated numerically. An asymmetric thermal boundary condition, particularly at upper plate insulated and lower plate with constant surface temperature is solved and analyzed in details graphically. Results reveal that effect of temperature field is strongly dependent on Brinkman number, while the thermal diffusion rate on the heat induced relies on the Prandtl number. The angular frequency has influence on the temperature field gradient.**

Keywords—Viscous dissipation, microchannel, oscillating fluid flow, Brinkman number, Prandtl number

I. Introduction

In view of the striking rise of microchannel application, such as microheat exchangers used in electronic component, genetic and biomedical engineering, the fundamentals on heat transfer are a vital criteria for the usage reliability [1,2]. The gap size for microchannel are often assorted in the range 10 $\mu m \leq D_h \leq 200\ \mu m$, where D_h is known as hydraulic diameter within the microchannel [3–5].

The viscous dissipation effect is often omitted in the heat transfer analysis in the conventional channel, unless for extreme viscosity at a comparatively high speed [6]. For fluid flow in microchannel and microtubes, the effect of viscous dissipation will generate significant heat [7–10]. Ignoring the viscous dissipation effect under these circumstances could affect the accuracy of analysis on fluid flow and heat transfer[7] .

Oftentimes, studies on viscous dissipation effect theme with hydrodynamic boundary conditions such as pressure driven, Couette, Couette-Poiseuille or stationary [11–18]. There is a recent study reviews numerically on the viscous dissipation effect on microchannel flows induced by an oscillatory lower plate [19].

As observed in Table I, it is found that the reported study has missed a set of mixed thermal boundary condition, for which upper boundary insulated and lower with constant surface temperature. This then become the motivation for present study. The present study intends to fill up the knowledge gap using a numerical method.

Table 1. Thermal boundary conditions for oscillatory microchannel.

	Thermal Boundary Conditions	
	Lower plate	***Upper plate***
Tso *et al.* [19]	Constant surface temperature	Constant surface temperature
	Insulated	Insulated
	Insulated	Constant surface temperature
Present study	Constant surface temperature	Insulated

II. Problem Statement

A. Hydrodynamic Profile

Figure 1 displays the schematic diagram for the viscous fluid flow that driven by the lower oscillatory plate, while upper is kept at stationary. The microchannel is distanced with W apart from the boundaries.

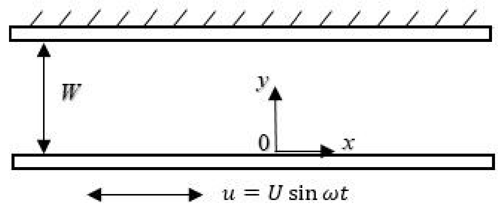


Fig. 1. Schematic diagram for the fluid flow induced by lower oscillatory plate.

The general conservation of momentum in the typical symbols for incompressible and laminar Newtonian flow is as the following [20]:

$$\rho\left(\frac{\partial\vec{u}}{\partial t}+\vec{u}\cdot\nabla\vec{u}\right) = -\nabla p + \mu\nabla^2\vec{u} + \rho\vec{F}. \quad (1)$$

This study consider infinite one-dimensional with unsteady flow, no pressure gradient and body force, the general equation in Eq. (1) is reduces to Eq. (2), constituting only the unsteady flow term and viscous term.

$$\frac{\partial u}{\partial t} - \nu\frac{\partial^2 u}{\partial y^2} = 0\,. \quad (2)$$

The hydrodynamic boundary and initial conditions considered are:

$$u(0,t) = U\sin(\omega t)\,, u(W,t) = 0 \text{ and } u(y,0) = 0. \quad (3)$$

The hydrodynamic profile in Eq. (2) has been reported numerically in the recent study. This then will be carry forward to investigate the thermal characteristic of the fluid.

B. Thermal Profile

The general conservation of energy for incompressible and laminar Newtonian flow is as the following [21]:

$$\rho c_p\left(\frac{\partial T}{\partial t}+\vec{u}\cdot\nabla T\right) = k\nabla^2 T + \mu\boldsymbol{\Phi} + q'''. \quad (4)$$

Since present study investigate uniquely the viscous dissipation effect, the heat source is omitted from Eq. (4). The convective term can be neglected in microchannel because the minute characteristic length of *W,* as shows in Fig. 1, alluded to a low Reynold number. Thus, Eq. (4) leaves only the heat storage, conduction and viscous dissipative terms in Eq. (5).

$$\rho c_p\frac{\partial T}{\partial t} = k\left(\frac{\partial^2 T}{\partial y^2}\right) + \mu\left(\frac{\partial u}{\partial y}\right)^2. \quad (5)$$

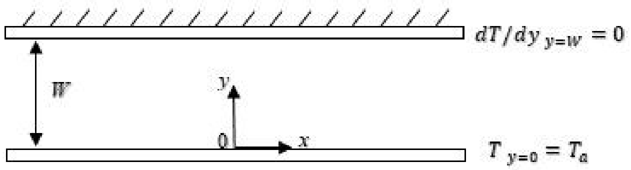


Fig. 2. Schematic diagram for the thermal boundary conditions in the microchannel oscillatory flow.

The boundary and initial conditions considered for the energy equation is illustrated in Fig. 2, where

$$T(0,t) = T_a\,, \frac{dT}{dy}(W,t) = 0 \text{ and } T(y,0) = T_a. \quad (6)$$

The upper stationary is insulated while the lower oscillating plate is kept at constant surface temperature. The initial temperature of the fluid is assumed to be T_a, an arbitrary temperature ($T_a = T_1$).

C. Dimensionless Form of the Equations

The momentum and the thermal equations will be recast into dimensionless form as to increase the efficiency in the analysis stage. The dimensionless terms utilise the typical symbols in conservation equations and are defined as:

$$u^* = \frac{u}{U}, \quad y^* = \frac{y}{W}, Pr = \frac{\nu}{\alpha}, \quad (7)$$

$$T^* = \frac{T-T_1}{T_m-T_1}, \quad Br = \frac{\mu U^2}{k(T_m-T_1)},$$

$$t^* = \frac{\nu t}{W^2}, \text{ and } \omega^* = \frac{\omega W^2}{\upsilon},$$

where u^* represent the fluid velocity, y* is the distance in y-axis, *Pr* is the Prandtl number, T^* is the fluid temperature, *Br* is the Brinkman number, t^* is the time dependent term, and ω^* is the angular frequency. T_m is the mean fluid temperature and T_1 is an arbitrary temperature which will not be equal to T_m.

Thus, the dimensionless forms of Eq. (2), Eq. (3), Eq. (5) and Eq. (6) using the definition in Eq. (7) are:

$$\frac{\partial u^*}{\partial t^*} - \frac{\partial^2 u^*}{\partial y^{*2}} = 0\,. \quad (8)$$

$$Pr\left(\frac{\partial T^*}{\partial t^*}\right) - \frac{\partial^2 T^*}{\partial y^{*2}} = Br\left(\frac{\partial u^*}{\partial y^*}\right)^2. \quad (9)$$

$$u^*(0,t^*) = \sin(\omega^* t^*)\,, u^*(1,t^*) = 0 \text{ and } u^*(y^*,0) = 0. \quad (10)$$

$$T^*(0,t^*) = 0\,, \frac{dT^*}{dy^*}(1,t^*) = 0 \text{ and } T^*(y^*,0) = 0. \quad (11)$$

III. RESULTS AND DISCUSSION

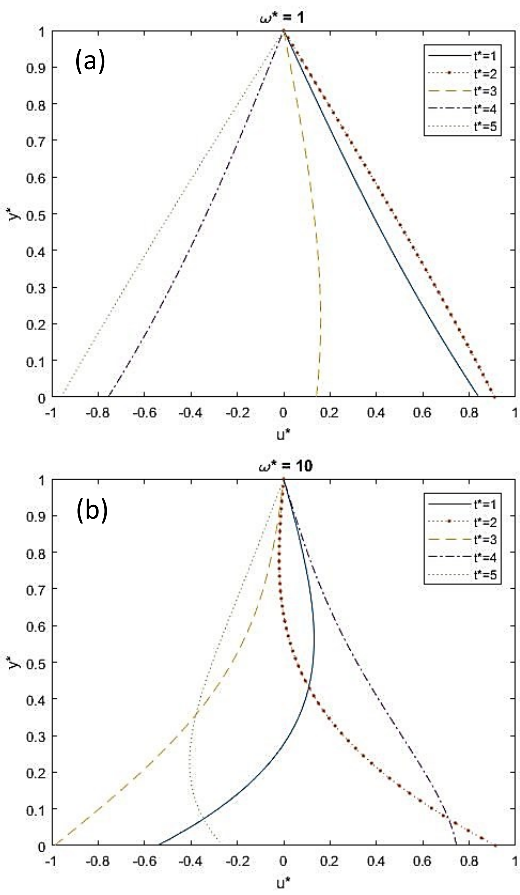


Fig. 3. Two selected velocity profiles in the microchannel induced by the oscillatory flow at (a) $\omega^* = 1$ and (b) $\omega^* = 10$.

The dimensionless form of equations for Eq. (8) and Eq. (9) are solved numerically through Matlab software. Since the model to be solved is a boundary value problem, the initial guesses for the slopes of the dependent variables are required. The optimum time step to solve this model at the selected ranges after trial of computations, is found to be $\Delta t^* = 0.02$.

The range of the dimensionless parameters are set at, $1 \leq Br \leq 10$, $0.5 \leq Pr \leq 5$, $1 \leq \omega^* \leq 10$ and $1 \leq t^* \leq 5$.

The varying in Brinkman number reflects on the viscous dissipation effect and Prandtl number indicates the effect on ratio of momentum diffusivity over thermal diffusivity. ω^* will reveal on the effect of angular frequency of the oscillatory plate.

A. Velocity Profiles

The velocity profiles display in Fig. 3 had been reported in a recent study. It is being imported here for illustrative purpose, and the results obtained will be extended to solve the temperature profile, using Eq. (9).

At a lower ω^*, the flow profile tends to behave like a linear velocity gradient and steeper when the ω^* increases. This reflects the fluid inertia towards the flow profile due to the viscous effect driven by the oscillatory lower plate.

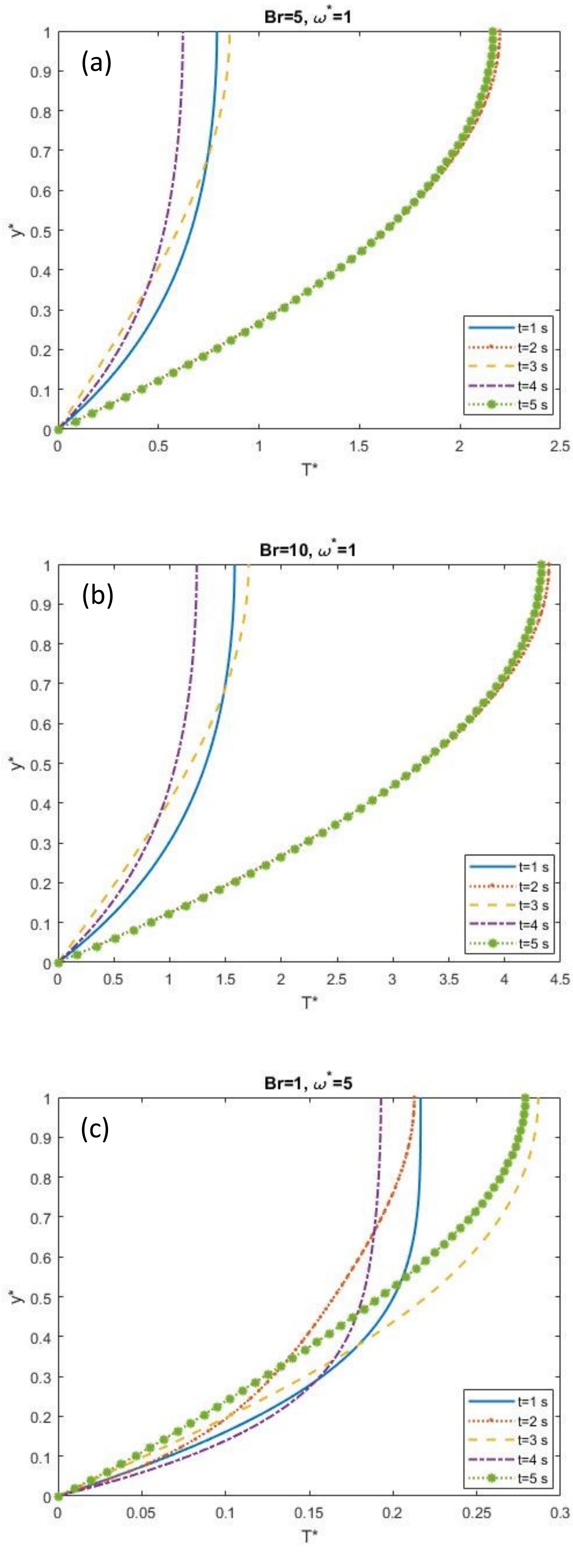
B. Temperature profiles

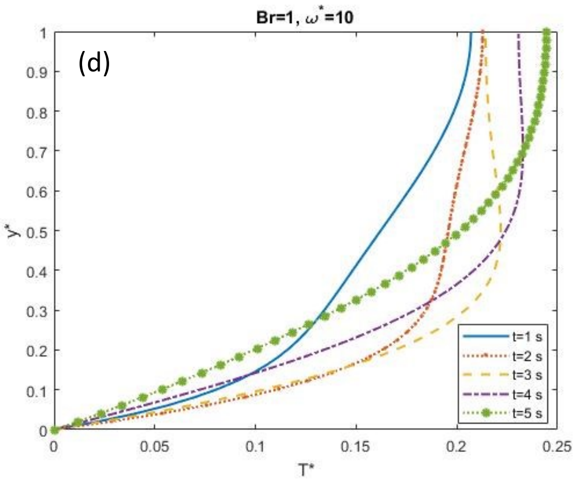
Figure 4 reveals the fluid temperature profiles at the respective dimensionless parameters with respect to selected dimensionless times.

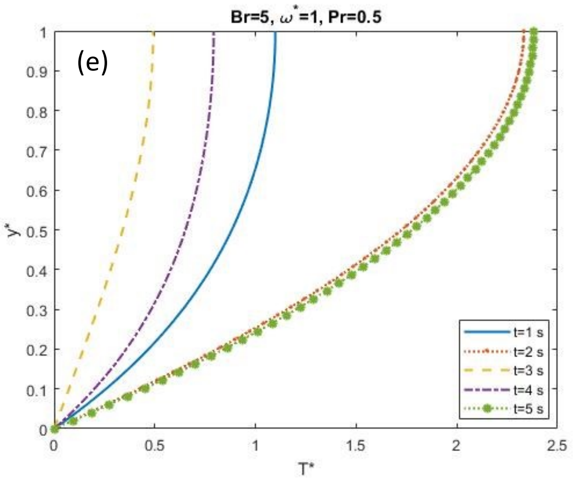
Figures 4(a) and 4(b) brings to light the viscous dissipation effect induced by the oscillatory flow. It is seen that the temperature profiles are oscillating with time as heat is allowed to diffuse out of the microchannel through the lower plate. When the Br is double, the temperature profiles are seen to remain but double in the numerical value. For example, at $y^* = 1$ and $t^* = 2$, for $Br = 5$, the $T^* \approx 2.2$ and for $Br = 10$ the $T^* \approx 4.4$, as in Figs. 4(a) and 4(b), respectively. It implies that the rise in fluid temperature is directly proportional to Br.

The effect of angular frequency is compared in Figs. 4(c) and 4(d). It is clear that the temperature gradient is getting precipitous as the ω^* increases. It is noteworthy that the peak temperature at certain time occurred somewhere within the microchannel at high ω^*. For example, in Fig. 3(d) at $t^* = 3$, the peak occurs near $y^*=0.4$. At low ω^*, say for $\omega^*=1$ cases, the peak temperature is seen to be located at the top plate always.

As observed from Figs. 4(a), 4(e) and 4 (f), the temperature profiles with lower value of Pr appear to be higher, compared to the higher one, because the heat diffusion is dominant at lower Pr. The slow momentum diffusion is prone to retain more heat generated caused by the viscous dissipation. For example, at $y^* = 1$ and $t^* = 2$, for $Pr = 1$, the $T^* \approx 2.2$, for $Pr = 0.5$ the $T^* \approx 2.4$ and for $Pr = 5$ the $T^* \approx 1.0$ as in Figs. 4(a), 4(e) and 4(f), respectively.







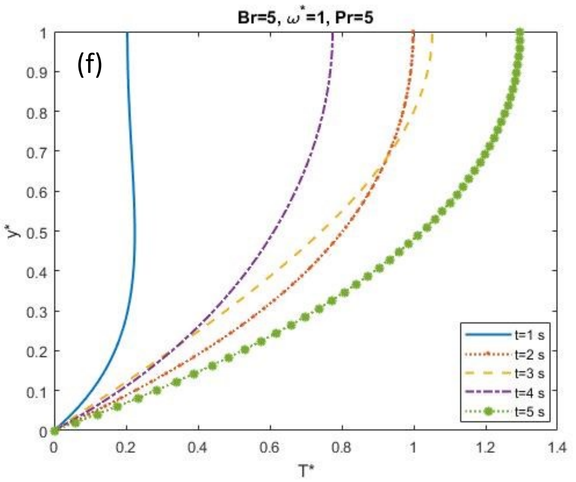


Fig. 4. Fluid temperature profiles in the microchannel induced by the oscillatory flow at (a) $Br = 5$, $\omega^* = 1$, $Pr = 1$, (b) $Br = 10$, $\omega^* = 1$, $Pr = 1$, (c) $Br = 1$, $\omega^* = 5$, $Pr = 1$, (d) $Br = 1$, $\omega^* = 10$, $Pr = 1$, (e) $Br = 5$, $\omega^* = 1$, $Pr = 0.5$, and (f) $Br = 5$, $\omega^* = 1$, $Pr = 5$.

IV. CONCLUSION

The present study reports the heat generation caused by the viscous dissipation effect induced by the oscillatory lower plate within a microchannel, particularly for a mixed thermal boundary condition, where upper plate is insulated and the lower plate is kept at constant surface temperature. Graphical analysis are done on the effects of Br, ω^*, Pr at the selected t^*. Results show that the induced heat generation is significantly dependent on the Br as the temperature field increases with Br. The thermal field gradient of the flow gets steeper as ω^* increases. The lower Pr, where the thermal diffusion is dominant, the more remnant heat is trapped in the microchannel, caused overall higher temperature field. The present results will be useful for verifying an analytical solution when it is available.

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