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Analytical Investigation of Nonlinear Dynamics of Soliton Transmission in Discrete System under Self-Induced Regime

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***Abstract*—This study investigates the soliton propagation in a one-dimensional discrete system characterized by the Discrete Nonlinear Schrödinger Equation (DNLSE). The DNLSE is a fundamental model in wave phenomena, encompassing a broad spectrum of physical systems ranging from optics to fluid dynamics. The analytical study employs the variational approximation (VA) method to thoroughly examine the process and essential parameters governing soliton evolutions, such as width, center-of-mass position, and linear and quadratic phase-front corrections are determined and graphically interpreted. The results show that an increase in linear phase-front correction corresponds to an increase in both the soliton's initial velocity and propagation distance.**

Keywords*—Discrete soliton, Nonlinear Schrödinger equation, Discrete system, Nonlinear partial differential equation, Variational approximation method.

I. INTRODUCTION

A discrete system is a system made up of a set of discrete points or lattices in which the values of the variables are often integers or other discrete values and they can only carry these exact values. The so-called discrete soliton, which is generated from a balancing effect of self-trapping nonlinearity and discrete diffraction mediated by the linear coupling of neighbouring sites, is a stable and localized wave that is practical in physical applications just as its continuous counterparts. At a particular time, the system's state is specified by the distinct variables at each site and any change in those variables may be influenced by the values at other sites [1]. Discrete systems are practically found in broad applicability such as in micromechanical cantilevers [2], Bose-Einstein Condensates (BEC) with a deep optical lattice [3, 4], electrical transmission lines [5] and Deoxyribonucleic acid (DNA) molecular chains [6, 7] where discrete solitons have been observed. Indeed, nonlinear optics was the first set of experimental studies that aroused significant intrigue in the research of discrete soliton, according to Kevrekidis [8].

Meanwhile, self-induced regime is a crucial concept in studying nonlinear wave equations, particularly within the discrete system of nonlinear Schrödinger equation (NLSE). It explains how solitons evolve in a discrete medium without external influences, driven by the interaction between the self-focusing effect of nonlinearity and spreading effect of dispersion phenomena. Researchers have focused on understanding the self-induced phenomena in wave field dynamics and control within nonlinear media [9] and weakly coupled optical waveguides [10].

This work focuses on the single discrete soliton propagation dynamics within self-induced regime, where its motion is primarily governed by the interplay of nonlinear and dispersive effects without the influence of external potential. The findings presented in this section lay the groundwork for understanding soliton behaviour in isolated systems, serving as a baseline for future studies involving perturbations or external influences.

II. THE MODEL OF MAIN EQUATION

The model of the main equation adopted for this study is based on the one-dimensional discrete NLSE with cubic and quintic terms, as modified from Balakin *et al.* [11]:

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$$i\frac{\partial \psi_n}{\partial t}+c\left(\psi_{n+1}+\psi_{n-1}-2\psi_n\right)+k\left|\psi_n\right|^2\psi_n \\ +g\left|\psi_n\right|^4\psi_n=0. \tag{1}$$

, where $\psi_n(t)$ denotes the complex wave function at a discrete site $n \in Z$ for time *t*. The coupling coefficient *c*, also referred to as the diffusion rate [12], quantifies the interaction strength between neighbouring lattice sites, while *k* and *g* represent the strengths of cubic and quintic nonlinearities, respectively. The real-valued parameters *c*, *k* and *g* remain constant as the lattice structures are equipped with identical sites and equal spacing between them.

The quantity $\left|\psi_n\right|^2$ may have different physical representations based on the systems applied. In particular, the quantity $\left|\psi_n\right|^2$ signifies the electric field intensity within the *n*-th waveguide in the context of waveguide arrays, studied by Lederer in 2008 [13], which can be seen in Fig. 1, whereas the term is associated with the density of on-site particle at the lattice site *n* in the discrete systems of BEC. Consideration is given to the positive coefficient of nonlinearities ($k, g > 0$), which indicates the nonlinear focusing behaviour in optics or attractive interactions among atoms in the condensate. This characteristic enables the system to exhibit bright matter-wave solitons that have a pulse-like shape.

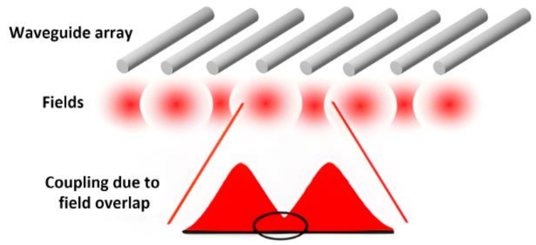

Fig. 1. An array of optical waveguides.

Two dynamical invariants known as the wave-field power (i.e. the norm) and the Hamiltonian (i.e. the energy) are conserved in the setting of Eq. (1) such that the power is given by

$$P=\sum_{n=-\infty}^{+\infty}\left|\psi_n\right|^2 \tag{2}$$

typically referred to as the norm of the system. In particular, the behaviour of the wave beams is greatly influenced by the value of *P* which acts as a controlling parameter for the entire system. Through deliberate manipulation of *P*, one can modulate the characteristics of the radiation to meet the desired outcomes for different applications effectively. The Hamiltonian is also expressed by

$$H=\sum_{n=-\infty}^{\infty}\left[\psi_{n+1}\psi_n^*+\psi_{n+1}^*\psi_n+\frac{1}{2}\left|\psi_n\right|^4+\frac{1}{3}\left|\psi_n\right|^6\right]. \tag{3}$$

These two conserved quantities are crucial in the dynamics of discrete soliton to ensure the consistency of its shape and amplitude over time without having radiative losses or dissipation. The next section discusses the variational analysis of the soliton's governing equation which produces the approximate systems of ordinary differential equations (ODEs) for the soliton variational parameters.

III. VARIATIONAL APPROXIMATION METHOD

This study emphasizes the use of the VA method as a key analytical tool for solving the nonintegrable Eq. (1), as numerical approaches may not fully describe the physical interpretation of the process. This method serves as the primary tool for examining the behaviour and interaction of the soliton wave beams scattering process, including their scattering dynamics within the cubic-quintic discrete NLSE. This investigation proceeds in the case of static states of the one-dimensional system Discrete NLSE. Initially, the main Eq. (1) is tackled using the VA method to derive analytical solutions for the evolution of soliton parameters, essential for characterizing the soliton scattering phenomenon.

VA method stands out as a key theoretical approach for studying non-integrable equations with soliton characteristics, dating back to its initial application by Anderson (1983) [14]. Anderson first employed this method to examine soliton behaviour within a significantly perturbed NLSE, particularly in nonlinear optics.

Essentially, the VA method provides approximate solutions grounded on particular assumptions requiring an appropriate trial function (ansatz) as the initial guess for the wave function. In this work, the wave function is initially assumed to resemble a Gaussian function profile in [15],

$$\psi(x,t)=\sqrt{\frac{P}{a\sqrt{\pi}}} \\ \times\exp\left(-\frac{(x-x_0)^2}{2a^2}+i\gamma(x-x_0)+i\beta(x-x_0)^2\right). \tag{4}$$

In this context, $a(t)$, $x_0(t)$, $\gamma(t)$ and $\beta(t)$ denote the effective width, center-of-mass position, and the linear and quadratic phase-front corrections of the soliton, respectively. The main partial differential equation (PDE) is then converted into an ordinary differential equations system which describes the above soliton parameters' evolution.

IV. RESULTS AND DISCUSSION

A. Variational Analysis of PDE

In the framework of nonlinear wave equations, it is beneficial to employ the variational approach to provide an approximate depiction of the evolution of the wave beams, particularly in scenarios where traditional analytical techniques fall short. Below is the Lagrangian of the system for the Cubic-Quintic DNLSE in Eq. (1) as the initial parameter for this approach,

$$L=\sum_{n=-\infty}^{\infty}\mathrm{L}_n=\sum_{n=-\infty}^{\infty}\frac{i}{2}\left(\psi_n\frac{\partial\psi_n^*}{\partial t}-\psi_n^*\frac{\partial\psi_n}{\partial t}\right)-H$$

$$=\sum_{n=-\infty}^{\infty}[\frac{i}{2}\left(\psi_n\frac{\partial\psi_n^*}{\partial t}-\psi_n^*\frac{\partial\psi_n}{\partial t}\right) \quad (5)$$

$$-c\left(\psi_{n+1}\psi_n^*+\psi_{n+1}^*\psi_n-2|\psi_n|^2\right)-\frac{k}{2}|\psi_n|^4-\frac{g}{3}|\psi_n|^6]$$

, where L_n denotes the Lagrangian density. Through the application of the Poisson summation formula applied to the continuous argument function *F(x)*, i.e.,

$$\sum_{n=-\infty}^{\infty}F(n)=\int_{-\infty}^{\infty}F(x)\sum_{n=-\infty}^{\infty}\exp(2\pi inx)dx \quad (6)$$

the Lagrangian presented in Eq. (5) is restructured into a more practical form given by

$$L=\sum_{n=-\infty}^{\infty}\mathrm{L}_n=\int_{-\infty}^{\infty}\mathrm{L}\sum_{n=-\infty}^{\infty}e^{2\pi inx}dx=\sum_{n=-\infty}^{\infty}\int_{-\infty}^{\infty}\mathrm{L}e^{2\pi inx}dx$$

$$=\sum_{n=-\infty}^{\infty}\int_{-\infty}^{\infty}[\frac{i}{2}(\psi(x,t)\frac{\partial\psi^*(x,t)}{\partial t}-\psi^*(x,t)\frac{\partial\psi(x,t)}{\partial t}) \quad (7)$$

$$-\frac{k}{2}|\psi(x,t)|^4-\frac{g}{3}|\psi(x,t)|^6-c(\psi(x+1,t)\psi^*(x,t)$$

$$+\psi^*(x+1,t)\psi(x,t)-2|\psi(x,t)|^2)]e^{2\pi inx}dx.$$

which further facilitates calculation process within this continuous approximation approach, considering the discrete nature of the system. The wave function, $\psi(x,t)$ in Eq. (7) depends on the continuous variable *x* with the time evolution variable *t*. In this case, it is assumed that the initial pulse is taken in the form of a Gaussian function with time dependent parameters as in Eq. (4), is adopted as the trial function.

Then, the effective Lagrangian is calculated with spatial integration of the above Lagrangian density in Eq. (7) such that $L=\sum_{n=-\infty}^{\infty}\int_{-\infty}^{\infty}\mathrm{L}e^{2\pi inx}dx$ and simplify it into

$$L=\frac{P}{2}\sum_{n=-\infty}^{\infty}\begin{matrix}[-2n^2\pi^2a^4\frac{d\beta}{dt}-2\gamma\frac{dx_0}{dt}\\+a^2\left(\frac{d\beta}{dt}-2in\pi\left(2\beta\frac{dx_0}{dt}-\frac{d\gamma}{dt}\right)\right)]e^{n\pi(-n\pi a^2+2ix_0)}\end{matrix}$$

$$-\frac{kP^2}{2\sqrt{2\pi}a}\sum_{n=-\infty}^{\infty}e^{-\frac{1}{2}n\pi(n\pi a^2-4ix_0)}-\frac{gP^3}{3\sqrt{3}\pi a^2}\sum_{n=-\infty}^{\infty}e^{-\frac{1}{3}n\pi(n\pi a^2-6ix_0)}$$

$$-cPe^{-\frac{1}{4a^2}-\beta^2a^2+i\gamma}\sum_{n=-\infty}^{\infty}e^{-a^2(n^2\pi^2+2\beta n\pi)+i(2n\pi x_0-n\pi)}$$

$$-cPe^{-\frac{1}{4a^2}-\beta^2a^2-i\gamma}\sum_{n=-\infty}^{\infty}e^{-a^2(n^2\pi^2-2\beta n\pi)+i(2n\pi x_0-n\pi)}$$

$$+2cP\sum_{n=-\infty}^{\infty}e^{n\pi(-n\pi a^2+2ix_0)}. \quad (8)$$

Equation (8) can be further simplified by combining the fifth and sixth terms, leading to the reduced lagrangian below,

$$L_c=\frac{Pa^2}{2}\frac{d\beta}{dt}-P\gamma\frac{dx_0}{dt}-\frac{kP^2}{2\sqrt{2\pi}a}$$

$$-\frac{gP^3}{3\sqrt{3}\pi a^2}-2cP\cos\gamma e^{-\frac{1}{4a^2}-\beta^2a^2}+2cP. \quad (9)$$

Equation (9) can be modified to produce a set of motion equations describing the ansatz parameter evolutions. Particularly, these equations are generated using the Euler-Lagrange equation

$$\frac{d}{dt}\left(\frac{\partial L}{\partial q_t}\right)-\frac{\partial L}{\partial q}=0, \quad (10)$$

in order to obtain the equations for the parameters. Upon formulating the equation for each parameter, a set of motion equations describing each ansatz parameter evolution is produced. The equations are represented as below:

$$\frac{da}{dt}=4\beta ac\cos\gamma e^{-\frac{1}{4a^2}-\beta^2a^2}, \quad (11)$$

$$\frac{dx_0}{dt}=2c\sin\gamma e^{-\frac{1}{4a^2}-\beta^2a^2}, \quad (12)$$

$$\frac{d\gamma}{dt}=0, \quad (13)$$

$$\frac{d\beta}{dt}=-\frac{kP}{2\sqrt{2\pi}a^3}-\frac{2gP^2}{3\sqrt{3}\pi a^4}+\frac{1-4\beta^2a^4}{a^4}c\cos\gamma e^{-\frac{1}{4a^2}-\beta^2a^2}, \quad (14)$$

, where Eqs. (11) – (14) represent the evolution of the soliton width, center-of-mass position, linear and quadratic phase-front corrections, respectively, during the soliton propagation. This system of variational equations for soliton parameters is obtained to describe the soliton scattering by external potential.

Analysis of the above system indicates that Eq. (12) represents the velocity of the soliton throughout the propagation process. This connection follows from the fundamental definition of velocity as the time derivative of position. Therefore, the following relationship is deduced,

$$v=\frac{dx_0}{dt}=2c\sin\gamma e^{-\frac{1}{4a^2}-\beta^2a^2}. \quad (15)$$

Equation (13) conveys the linear phase-front correction's independence on coordinate *t*, indicating consistency throughout the propagation process such that $\gamma=\gamma_0$. Equations (11) – (14) are then interpreted numerically to observe the behaviour of soliton's propagation through the system in the absence of external potential. The observation involves configuring the waveguide lattice with site $n=200$ and monitoring the process over a time span of $t=300$. The numerical setup assumes a coupling strength of $c = 1$ between adjacent lattice sites. Equal values are also assigned to the nonlinearity coefficients with $k = g = 1$. The soliton parameters are initialized as $a(0) = 3$, $x_0(0) = 50$, $\beta(0) = 0$ to minimize the number of free variables in the numerical simulations. The value of parameter γ is adjusted to observe its influence on power, initial velocity and the propagation process as a whole.

The scattering results in the system of discrete cubic-quintic NLSE in the self-induced regime are illustrated in Fig. 2. Initial analysis in Fig. 2(a) indicates that the soliton remains stationary over time at the lattice site $n = 50$ when $\gamma = 0$. Subsequent observation in Fig. 2(b) reveals a notable change in the soliton's behaviour as the value of γ is incrementally increased to 0.1. In this condition, the soliton demonstrates the ability to propagate with a velocity of $v = 0.194197$ and a corresponding power of $P = 1.28114$ whereby the center-of-mass position surpasses $n = 100$ at the end of the observed time interval $t = 300$.

Further observation, as depicted in Fig. 2(c), shows a significant advancement in soliton motion when γ is further increased to 0.5. Specifically, the center-of-mass position traverses the entire lattice site, reaching $n = 200$ within a considerably shorter time at $t = 160$. Here, the soliton's velocity notably increases to $v = 0.932583$, accompanied by a slight decrease in power to $P = 1.15379$. The recorded patterns emphasize a progressive increment in soliton's velocity and propagation distance as the linear phase-front correction parameter γ is elevated from 0.1 to 0.5. Consequently, the time required for the soliton to travel the same distances is decreased. Despite the variations in velocity and power, the soliton's width remains unchanged throughout the entire propagation path in all scenarios, as visually depicted in the left panels of Fig. 2. This suggests that the soliton maintains its stability and is freely propagating.

In this study, the variables such as velocity, power, time and linear phase-front correction are expressed in dimensionless (scaled) units, which is standard practice in variational analysis of discrete NLSEs. The scaling enhances the universality of the results, allowing them to be interpreted across a broad range of physical systems, such as nonlinear optical lattices and Bose-Einstein condensates.

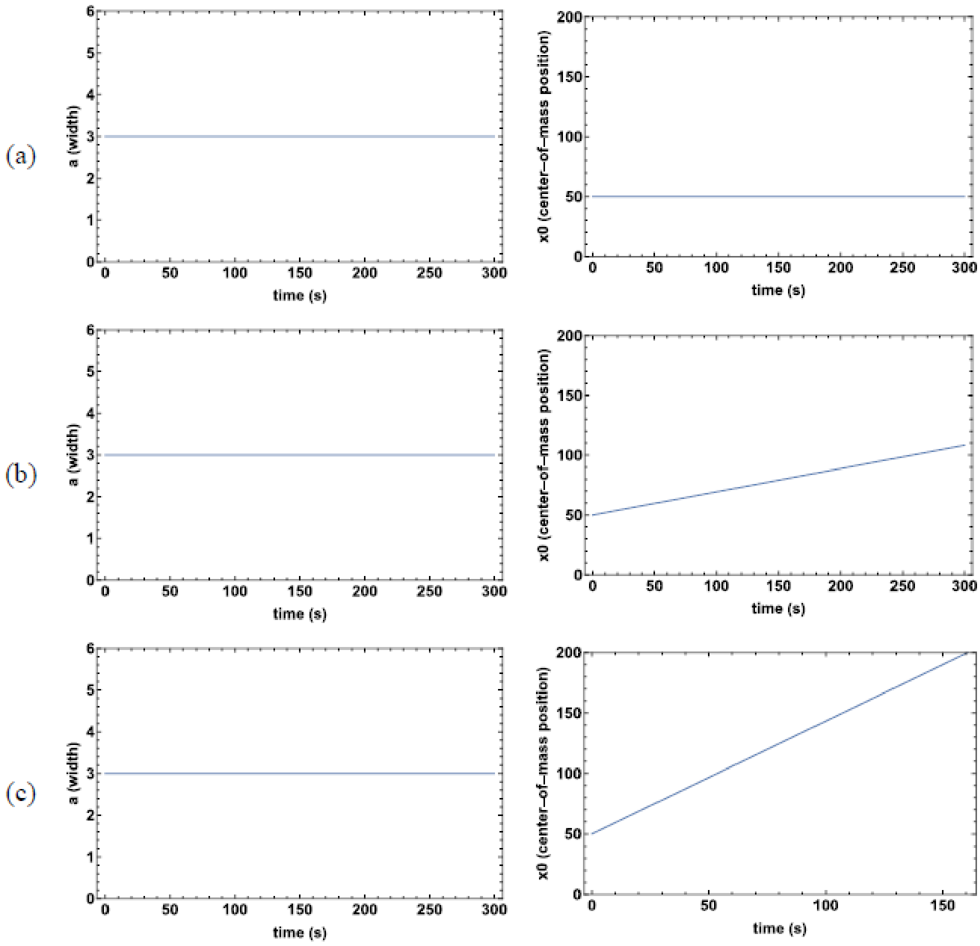

Fig. 2. The dynamics of the soliton width (left panel) and center-of-mass position (right panel) over time *t* described by the ODE systems for Eqs. (11) and (12). Parameters used are (a) $\gamma = 0$, (b) $\gamma = 0.1$ and (c) $\gamma = 0.5$.

The scaling procedure is grounded on characteristic system parameters, including the inter-site lattice spacing, coupling strength, and nonlinear interaction coefficients. By adopting these dimensionless formulations, the results remain independent of specific physical units, thereby facilitating more generalized and scalable interpretations of the soliton dynamics under investigation.

The study has revealed that the soliton's velocity and propagation distance increased with linear phase-front correction γ. The findings highlight the importance of VA approaches for faster results and a deeper perception of the system's physical aspects. The findings also shed light on understanding and harnessing wave behaviour across different disciplines, paving the way for innovative applications in those leveraging soliton-driven systems.

V. CONCLUSION

The behaviour of soliton propagation within the self-induced regime was investigated in the framework of the discrete cubic-quintic NLSE through the VA method. The study took place by considering different initial values for the soliton parameter of linear phase-front correction γ obtained from the variational analysis to see the impacts on soliton propagation. The findings showed that the soliton exhibited static behaviour when γ is zero, considering the initial velocity being zero. An increase in γ prompted the soliton to move with velocity increasing proportionally.

The observed soliton dynamics are consistent with trends reported in related studies on discrete nonlinear Schrödinger systems using variational methods by Anderson in 1983 [14] and Lederer *et al.* in 2008 [13]. The VA method has proven successful since its first application by Anderson. In his paper, he analyzed the soliton evolutions in the NLSE within the context of optical fibers by utilizing the Gaussian trial function and a procedure based on Ritz optimization. Later, [16 - 21], among others, utilized the same approach to investigate the interaction of the continuous NLSE solitons in the presence of an external potential. The VA method enables the computation of approximate solutions for essential soliton parameters such as the width, amplitude, center-of-mass position, nonlinear frequency chirp and other parameters. These parameters are instrumental in providing insights into the wave propagation dynamics, thus offering the opportunity for in-depth analysis of soliton scattering phenomena. Although direct numerical simulations of the full discrete NLSE were not included in this paper, the VA method's results are qualitatively aligned with findings from such simulations in previous literature. A more detailed quantitative comparison with numerical integration of the original PDE system is planned for future work to validate the accuracy further.

The discrete system of nonlinear equations is particularly significant to our understanding and modelling of a wide range of physical phenomena. The behavior of soliton propagation within the self-induced regime was investigated through the framework of the discrete cubic-quintic nonlinear Schrödinger equation using the variational approximation (VA) method. By considering various initial values for the linear phase-front correction parameter obtained via VA, the study revealed a clear correlation between this parameter and the soliton's initial velocity and propagation distance. The results demonstrated that an increase in the linear phase-front correction leads to a corresponding increase in the soliton's speed and distance traveled, while maintaining its shape and stability throughout the propagation.

Beyond the theoretical insights, the findings also offer a meaningful practical implication. In the context of optical waveguide arrays, where solitons represent localized optical pulses, the ability to modulate the linear phase-front correction allows for dynamic control over pulse velocity and localization. This capability is fundamental for the realization of optical switching and signal routing functions in photonic devices. Consequently, the outcomes of this research contribute not only to the understanding of nonlinear wave dynamics in discrete media but also to the advancement of nonlinear optical circuit design and soliton-based technologies in integrated photonic systems.

Furthermore, the study on the numerical simulations of discrete systems utilizing the discrete nonlinear equations forms the basis for numerical simulations of complex systems. In particular, researchers are able to explore the behaviour of nonlinear systems, study bifurcations, and analyze the stability of solutions in a computationally efficient manner, leading to advancements in technology and scientific understanding. For instance, Lazar *et al.* [22] had investigated the Pacemaker and Implantabel Cardioverter Defibrillator (ICD) Troubleshooting advanced technology where the different formations of continuous and discrete systems are presented.

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