Journal of Informatics and Web Engineering

Vol. 4 No. 3 (October 2025) eISSN: 2821-370X

Implementation of Conjugate Gradient Method for Estimating Inflation Rate in Malaysia

Shin Yi Wong[1], Nur Syarafina Mohamed[2*], Norhaslinda Zullpakkal[3]

[1,2]Department of Mathematical Sciences, Faculty of Science, Universiti Teknologi Malaysia, 39/2, Jalan Ihsan, 80990, Johor Bahru, Malaysia.

[3]Faculty of Computer and Mathematical Sciences, Universiti Teknologi MARA Terengganu Branch Kuala Terengganu Campus, Terengganu, Malaysia.

**corresponding author: (nursyarafina@utm.my; ORCiD: 0000-0002-8572-2234*)

Abstract - Optimisation methods are valuable for making decisions and identifying the most suitable alternative based on a given objective function. One of the mathematical optimisation methods is Conjugate Gradient (CG) method which is commonly used to solve large-scale unconstrained optimisation systems with less storage space. Recently, various optimisation methods have been studied and used in economics estimating. However, just a few studies have predicted inflation rate using modified CG method. Random initial points are tested on New Three-Terms (NTT) which are modified Rivaie-Mustafa-Ismail-Leong (RMIL+) and Umar Mustapha Waziri (UMW) CG method with ten optimisation test functions suggested by Andrei using MATLAB. NOI and CPU time obtained are compared by performance ratio of Dolan and More. NTT CG method stands out as the best performance. Data set of year 2010 until 2022 from Department of Statistics Malaysia (DOSM) is transformed into Optimisation problems to be solved. Estimated results of Least Square Conjugate Gradient (LSCG) are based on NTT CG and LS both for linear and quadratic models. Relative errors for LSCG, Least Square (LS) and Trendline Method are calculated. Linear LS is shown as the most suitable to estimator in inflation rate in Malaysia as it yields the least relative error compatible with the linear LSCG and Trendline Method that produce similar relative error in estimating inflation rate in Malaysia.

Keywords—Three-Terms Conjugate Gradient Method, Data Estimation, Least Square Method, Trendline Method, Inflation Rate.

Received: 21 May 2025; Accepted: 27 August 2025; Published: 16 October 2025

1. INTRODUCTION

Optimisation methods play an important role in guiding decision-making processes. [1]. These methods could provide the optimum value which is desired. Optimisation methods are applied in many fields such as economics, engineering, and mechanics. These problems are described by an objective function while the optimisation methods are used to choose the best alternative of the given objective function [2]. One of the optimisation methods commonly used to solve real-world problems is the CG method. The CG method is an iterative optimisation algorithm designed to effectively solve large-scale linear equations or minimizing quadratic functions. Its relevance to economics arises in computational economic modelling, where economists build large, complex mathematical models such as input-output

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analysis, portfolio optimisation, or market equilibrium simulations that require solving huge systems of equations. CG method enables faster computation with less memory usage compared to direct methods, making real-time or large-scale economic analysis more practical. The CG method is one of the gradient-based methods that use the gradients of the problem functions to find the optimum point. It is known as an effective method which could be applied to both linear and nonlinear systems and solve large scale problems raised in statistics nonparametric, portfolio selection and image restoration problems [3] with less storage space required [4]. Many researchers have modified and improved the original scheme of CG method that can be classified into Hybrid [5], Spectral [6], Scaled [7] and Three-Terms [8] CG methods. These CG methods have been applied widely in solving regression analysis [9-12] . Recently, Ibrahim et al. applied Three-term CG (TTCG) to solve image restoration problem [13] while Khudhur et al. improved CG method to remove noise from images [14]. Ibrahim et al. introduced TTCG [15] for training artificial neural network in accurate heart disease prediction. This study is focused on TTCG method and is going to be implemented as one of the data estimator tools.

The inflation rate of a region could be also predicted by using modified CG method but there are just a few studies that have predicted the inflation rate of a region by using modified CG method. For the example, a novel TTCG method has applied on inflation rate in Nigeria from 2010 to 2018 by [16] while Wanto et al. modified Fletcher-Reeves CG method to predict the rate of inflation or deflation in Pematangsiantar [17]. Basically, the inflation rate is the average rate of prices increase of products or services. It is a good measure of the changes in goods and services prices in the market. Inflation has brought a series of influences to the country and citizens. It is also always related to a series of economic and social issues such as economic growth, unemployment rate, exchange rate, criminal rate and foreign direct investment inflows [18]. Generally, the aim of all countries is to keep controlling the inflation rate and maintaining a low inflation rate. Thus, the focus of this study is to apply the three-terms CG Method for estimating the inflation rate in Malaysia.

2. LITERATURE REVIEW

This study aims to implement the CG method as an estimation tool, with a particular emphasis on solving unconstrained optimisation problems using the TTCG approach. The experiment involves evaluating the performance of several selected methods to identify the most effective one. The best-performing method is then applied as a data estimator for inflation rate data. To validate the results, the proposed approach is compared with other estimation techniques – Linear Regression models, LS method and the Trendline method.

2.1 Unconstrained Optimisation Method

The CG method is one of the mathematical techniques used for solving unconstrained optimisation problems. Unconstrained optimisation could be considered in (1) as below,

$$\text{minimize } f(x) \qquad (1)$$

$$subject\ to\ x \in \mathbb{R}^n$$

and it is continuously differentiable [19]. It must satisfy global convergence and sufficient descent properties to ensure its efficiency. CG is an advancement of Steepest Descent (SD) method. The initial scheme has undergone modifications and advancements throughout the years. It is advantageous compared to the SD method since it requires low computational memory with lesser steps. Different CG methods are determined by different CG parameters [20].

2.2 Three-Term CG Method

One classification in CG methods is the TTCG method. The first general TTCG is proposed by Beale in 1972. The search direction,d_{k+1} and step size, α_{k+1} parameter of it is as shown in (2) and (3), [21].

$$d_{k+1} = -g_{k+1} + \beta_{k+1} d_k + \alpha_{k+1} d_t \qquad (2)$$

$$\alpha_{k+1} = \begin{cases} 0 & ,k = t+1 \\ \dfrac{g_{k+1}^T y_t}{d_t^T y_t} & ,k > t+1 \end{cases} \tag{3}$$

where d_t is restart direction.

2.3 Inflation Rate

Inflation rate in (4) is the average increase of the price level of the goods or services in a country, and it is measured by the change in the Customer Price Index (CPI). The accurate forecasting of the inflation rate is important for the bank, businessmen and customers in making countermeasures to cope with changes in economy [22].

$$Inflation\ rate = \frac{CPI_{x+1} - CP1_x}{CPI_x} \times 100 \tag{4}$$

2.4 Customer Price Index

The CPI in (5) is the average goods or services price acquired by household. It serves as a measuring tool for inflation, which frequently happens when the costs rising of labour and raw material [23]. It is computed at regular time periods to monitor the inflation rate.

$$CPI = \frac{\sum(CPI \times weightage)}{\sum weightage} \tag{5}$$

2.5 Test Function

Unconstrained optimisation test functions are used to evaluate and compare performance, including the convergency, accuracy and efficiency of the nonlinear optimisation algorithm [24]. These functions are designed to test the efficiency of a nonlinear optimisation function to find its minimum. Other than standard starting point, random starting points surrounding the standard starting points needed to be chosen to test the works and efficiency of the functions [25]. Hence, different types of test functions are studied to find out the most efficient TTCG method.

2.6 Performance Profile

Performance profile is widely used in running large test sets for benchmarking and evaluating the performance of several solvers [26]. The results of the comparison from the test functions are then represented by using a performance profile. The performance ratio in (6) is the performance comparison of problem p by solver s against the best performance solver [27]. It can be defined as

$$\tau_{p,s} = \frac{t_{p,s}}{\min\{t_{p,s}: s \epsilon S\}} \tag{6}$$

where $t_{p,s}$ is the computing time required to solve problem p by solver s. The cumulative distribution function representing the performance ratio in (7)

$$P_s(\tau) = \frac{1}{n_p} size\{p \epsilon P: r_{p,s} \leq \tau\} \tag{7}$$

The larger probability $P_s(\tau)$ are to be preferred as it represents the set of problems P is suitably large and likely to occur [28].

2.7 Linear Regression Method

Linear Regression Method is a statistical method that models the relationship between variables [29]. This method is employed by fitting the observed data into the linear equation which represents the relationship of a dependent variable and an independent variable, whereas multiple regression model includes several independent variables. The linear regression model is represented by $y_i = \beta_0 + \beta_1 x_0 + \beta_2 x_1 + \cdots + \varepsilon$ where y_i denotes the dependent variable, x_i signifies the independent variables, β_i indicates the coefficients of the regression equation and ε represents the error.

2.8 LS Method

As stated by [30], LS method is the best analytical technique for extracting information from a set of data. It is used in Linear Regression method to estimate the coefficients of the linear equation [31]. The objective of this method is to find the coefficients of the linear equation that could minimise sum of squared differences or relative error among the actual and estimation values.

3. RESEARCH METHODOLOGY

3.1 Three-Terms CG Method

The latest modified TTCG methods (8) – (10) are used in this research:

i. Modified RMIL+ coefficient which satisfies the sufficient descent property and global convergence under Strong Wolfe line search [32]

$$\beta_k^{RMIL+} = \begin{cases} \frac{g_k^T y_{k-1}}{\|d_{k-1}\|^2}, if\ 1 \le g_k^T g_{k-1} \le \|g_k\|^2 \\ 0, \quad otherwise \end{cases} \tag{8}$$

ii. UMW coefficient which is more effective and robust compared to traditional CG method such as FR and PRP [33]

$$\beta_k^{UMW} = \frac{\|g_k\|^2}{d_{k-1}^T \left(d_{k-1} - \frac{\|g_k\|}{\|g_{k-1}\|} g_k - g_k \right)} \tag{9}$$

iii. NTT CG coefficient which is always sufficiently descent and global convergence as well as outperformed in terms of NOI and CPU time when compared to TMRMIL, ISCG, CG Descent and Three CG methods [34]

$$\beta_k^{NTT} = \frac{g_{k+1}^T y_k}{{g_k}' g_k} \tag{10}$$

The algorithm of Three-Terms CG method is shown in Algorithm 1 as follows.

Algorithms 1

Step 1: Set $k = 0$, select the initial point x_0.
Step 2: Compute β_k based on RMIL+ (8), UMW (9) and NTT (10) coefficients.
Step 3: Compute the search direction using three term formula as in (2) and (3).
Step 4: Compute step size using Strong Wolfe line search.
Step 5: Updating $x_{k+1} = x_k + \alpha_k d_k$.
Step 6: Stop if $f(x_{k+1}) < f(x_k)$ or $\|g_k\| \le \varepsilon$. Otherwise, repeat Step 1 by $k = k + 1$.

3.2

3.3 Linear Regression

The most effective TTCG coefficient is implemented to estimate the inflation rate in Malaysia by using Linear Regression method. Linear Regression method is a statistical method that models the relationship between variables [35]. Linear regression model is represented by $y_i = \beta_0 + \beta_1 x_0 + \beta_2 x_1 + \cdots + \varepsilon$ where y_i denotes the dependent variable, x_i represents the independent variables, β_i represents the coefficients of the regression equation and ε indicates the error.

3.4 LS

The objective of LS method is fitting the observed data to complex models [36] to obtain the parameters of the estimation function that minimize the least square error or also known as sum squared error, E. The equation of least squared error, E is shown below where y_i denotes the actual data while $f(x_i)$ represents the fitting curve.

$$E = \sum_{i=1}^{n} e_i^2 = \sum_{i=1}^{n} (y_i - f(x_i))^2 \tag{11}$$

The fitting curve $f(x_i)$ could be represented by a straight line which is best fit to all data to show the trend of the data. Thus, the fitting curve could be represented by a straight-line formula.

$$f(x_i) = y = \beta_0 + \beta_1 x \tag{12}$$

However, the fitting curve $f(x_i)$ could also be represented by a parabola curve to ensure that there is a least error. The equation of parabola is given by

$$f(x_i) = y = \beta_0 + \beta_1 x + \beta_2 x^2 \tag{13}$$

Equation (11), the sum squared error can be updated by using equation (12) and (13) to form equations (14) and (15) respectively.

$$E = \sum_{i=1}^{n} \left(y_i - (\beta_0 + \beta_1 x)\right)^2 \tag{14}$$

$$E = \sum_{i=1}^{n} \left(y_i - (\beta_0 + \beta_1 x + \beta_2 x^2)\right)^2 \tag{15}$$

The necessary conditions for E to be minimum are

$$\frac{\partial E}{\partial \beta_0} = 0 \text{ and } \frac{\partial E}{\partial \beta_1} = 0$$

Thus, by differencing (14) and (15), it is transformed into the matrix form,

$$\begin{bmatrix} n & \sum_{i=1}^{n} x_i \\ \sum_{i=1}^{n} x_i & \sum_{i=1}^{n} x_i^2 \end{bmatrix} \begin{bmatrix} \beta_0 \\ \beta_1 \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^{n} y_i \\ \sum_{i=1}^{n} x_i y_i \end{bmatrix} \tag{16}$$

The values of β_0 and β_1 is calculated by using inverse matrix,

$$\begin{bmatrix}\beta_0\\ \beta_1\end{bmatrix} = \begin{bmatrix} n & \sum_{i=1}^{n} x_i \\ \sum_{i=1}^{n} x_i & \sum_{i=1}^{n} x_i^2 \end{bmatrix}^{-1} \begin{bmatrix} \sum_{i=1}^{n} y_i \\ \sum_{i=1}^{n} x_i y_i \end{bmatrix} \tag{17}$$

By using equation (15), the polynomial function is represented in matrix form,

$$\begin{bmatrix} n & \sum_{i=1}^{n} x_i & \sum_{i=1}^{n} x_i^2 \\ \sum_{i=1}^{n} x_i & \sum_{i=1}^{n} x_i^2 & \sum_{i=1}^{n} x_i^3 \\ \sum_{i=1}^{n} x_i^2 & \sum_{i=1}^{n} x_i^3 & \sum_{i=1}^{n} x_i^4 \end{bmatrix} \begin{bmatrix}\beta_0\\ \beta_1\\ \beta_2\end{bmatrix} = \begin{bmatrix} \sum_{i=1}^{n} y_i \\ \sum_{i=1}^{n} x_i y_i \\ \sum_{i=1}^{n} x_i^2 y_i \end{bmatrix} \tag{18}$$

The values of β_0, β_1 and β_2 are calculated using inverse matrix,

$$\begin{bmatrix}\beta_0\\ \beta_1\\ \beta_2\end{bmatrix} = \begin{bmatrix} n & \sum_{i=1}^{n} x_i & \sum_{i=1}^{n} x_i^2 \\ \sum_{i=1}^{n} x_i & \sum_{i=1}^{n} x_i^2 & \sum_{i=1}^{n} x_i^3 \\ \sum_{i=1}^{n} x_i^2 & \sum_{i=1}^{n} x_i^3 & \sum_{i=1}^{n} x_i^4 \end{bmatrix}^{-1} \begin{bmatrix} \sum_{i=1}^{n} y_i \\ \sum_{i=1}^{n} x_i y_i \\ \sum_{i=1}^{n} x_i^2 y_i \end{bmatrix} \tag{19}$$

After β_i is obtained, these values are substituted back into equations (11) and (12) to get the linear and quadratic Linear Square models. These models could be transformed into optimisation problems as shown below.

$$\min_{x \in R^n} f(x) = \sum_{i=1}^{n} \left(y_i - (\beta_0 + \beta_1 x)\right)^2 \tag{20}$$

$$\min_{x \in R^n} f(x) = \sum_{i=1}^{n} \left(y_i - (\beta_0 + \beta_1 x + \beta_2 x^2)\right)^2 \tag{21}$$

3.5 Trendline Method

Trendline is a line that shows the trend of the data in the chart [37]. The trendline of the data could be generated automatically by using the built-in function in Microsoft Excel after the data is transferred into the graph. The trendline could also be used to determine the parameter values of the Regression Model, β_i. Then, the model could be used to estimate the inflation rate in Malaysia.

3.6 Relative Error

Relative error is calculated after estimating value is obtained by fitting the data into the regression model to determine the accuracy of the model. The formula for calculating the relative error is given by (22).

$$relative\ error = \frac{|exact\ value - approximate\ value|}{|exact\ value|} \tag{22}$$

4. RESULTS AND DISCUSSIONS

The experiment evaluated the performance of the TTCG method by testing it on various types of test functions commonly used in mathematical optimisation. A subset of ten test functions shown in Table 1 were selected from [38] is used to test the efficiency of the optimisation methods. Each test function has different dimensions and characteristics. All the test functions are tested on TTCG method with selected coefficients, RMIL+, UMW and NTT by using MATLAB software. These methods are compared based on the number of iterations (NOI) and CPU time under Strong Wolfe line search to evaluate their numerical performance.

Table 1. List of Test Functions

No.	Test Function	Variables	Initial Points
1	Extended Freudenstein & Roth Function	2	(1,2), (3,3), (0,4), (5,6)
2	Six Hump Function	2	(5,5), (8,8), (18,18), (28,28)
3	Power Function	2	(2,2), (4,4), (12,12), (25,25)
4	Booth Function	2	(2,2), (8,8), (25,25), (31,31)
5	Raydan 1 Function	2	(2,2), (8,8), (15,15), (25,25)
		4	(2,..,2), (8,..,8), (15,…,15), (25,…,25)
6	FLETCHCR Function	2	(2,2), (4,4), (12,12), (25,25)
		4	(2,..,2), (4,..,4), (12,..,12), (25,..,25)
		10	(2,..,2), (4,..,4), (12,..,12), (25,..,25)
7	Sum Squares Function	2	(2,2), (8,8), (12,12), (25,25)
		4	(2,..,2), (8,..,8), (12,..,12), (25,..,25)
		10	(2,..,2), (8,..,8), (12,..,12), (25,..,25)
8	Extended White & Holst Function	2	(4,4), (8,8), (11,11), (15,15)
		4	(4,..,4), (8,..,8), (11,..,11), (15,..,15)
		10	(4,..,4), (8,..,8), (11,..,11), (15,..,15)
		500	(4,..,4), (8,..,8), (11,..,11), (15,..,15)
		1000	(4,..,4), (8,..,8), (11,..,11), (18,..,18)
9	Diagonal 4 Function	2	(2,2), (4,4), (7,7), (15,15)
		4	(2,..,2), (4,..,4), (7,..,7), (15,..,15)
		10	(2,..,2), (4,..,4), (7,..,7), (15,..,15)
		500	(2,..,2), (4,..,4), (7,..,7), (15,..,15)
		1000	(2,..,2), (4,..,4), (7,..,7), (15,..,15)
10	Extended Himmelblau Function	2	(2,2), (4,4), (10,10), (15,15)
		4	(2,..,2), (4,..,4), (10,..,10), (15,..,15)
		10	(2,..,2), (4,..,4), (10,..,10), (15,..,15)
		500	(2,..,2), (4,..,4), (10,..,10), (15,..,15)
		1000	(2,..,2), (4,..,4), (10,..,10), (15,..,15)

The results obtained are recorded and interpreted into the performance profile. The coefficient with highest $P_s(\tau)$ value where the curve dominates at the top left of the graph is the most efficient as it has the least NOI and CPU time which means it converges fastest.

From Figure 1 and Figure 2, the NTT coefficient is the most efficient method since it has the highest curve on the left side. It shows NTT yields the best performance in terms of NOI and CPU time, and it is followed by RMIL+ and

UMW. Based on the right curve, three methods can solve all the tested functions. Hence, the NTT method is going to be used as data estimator in estimating the inflations rate data.

The inflation rate in terms of CPI in Malaysia is estimated. From Table 2, the CPI data in Malaysia recorded annually from the year 2010 until the year 2022 based on the year 2010 is collected from DOSM. LS method, LSCG method and Trendline method are used and each of the methods is used to generate both linear and quadratic models. The most effective TTCG method, NTT coefficient, is implemented in the LSCG Method.

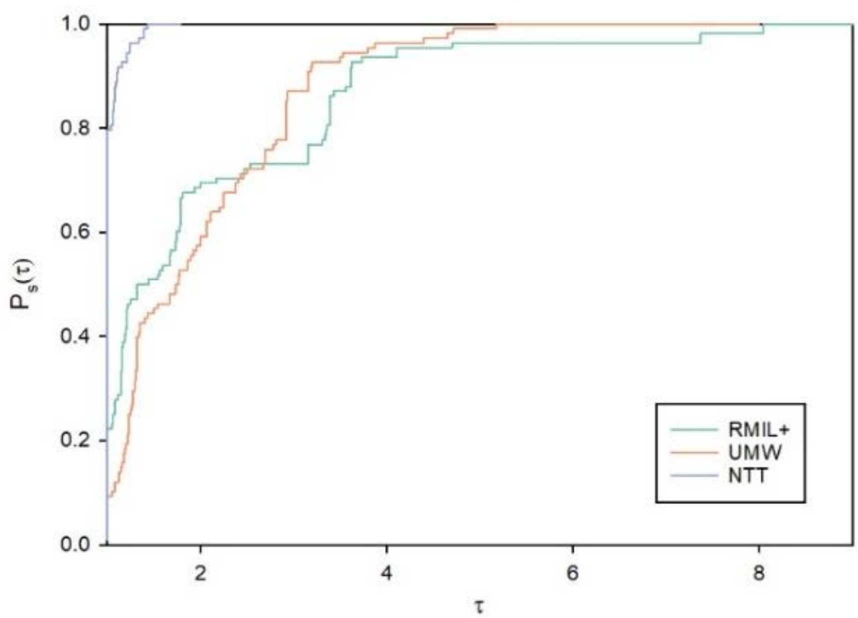

Figure 1. Performance Profile Based on NOI

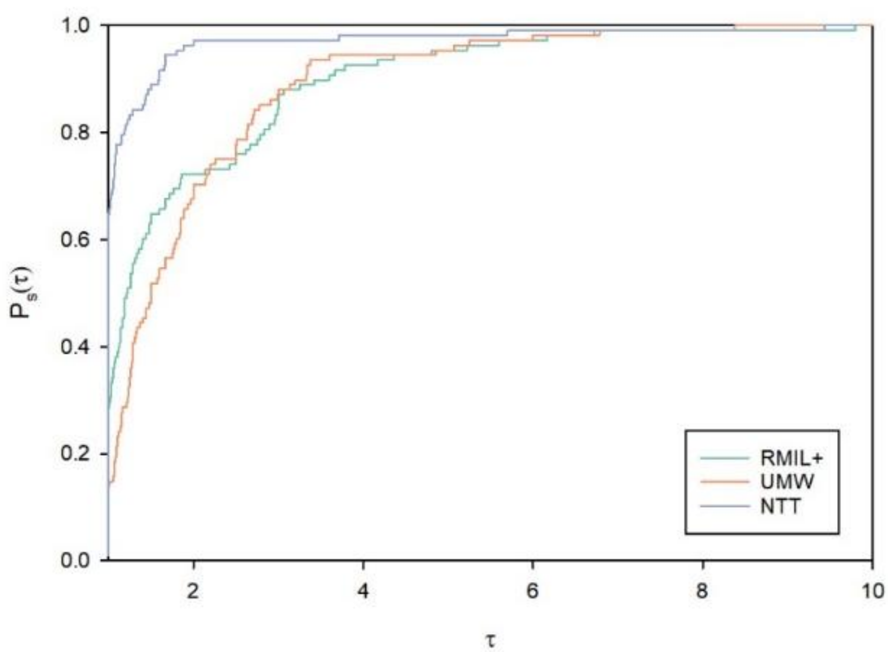

Figure 2. Performance Profile Based on CPU Time

Table 2. CPI Data

Number of observed data (x_i)	Years	CPI in Malaysia (y_i)	Ratio based on Year 2010 (%)
1	2010	100.0	1.000
2	2011	103.2	1.032
3	2012	104.9	1.049
4	2013	107.1	1.071
5	2014	110.5	1.105
6	2015	112.8	1.128
7	2016	115.2	1.152
8	2017	119.5	1.195
9	2018	120.7	1.207
10	2019	121.5	1.215
11	2020	120.1	1.201
12	2021	123.1	1.231
13	2022	127.2	1.282

The number of observed years is set as x_i while the inflation rate in Malaysia is set as y_i. The data of the year 2022 is reserved for prediction The linear and quadratic models are generated based on LS method by using (16) and (18) as shown below.

$$\begin{bmatrix} 12 & 78 \\ 78 & 650 \end{bmatrix} \begin{bmatrix} \beta_0 \\ \beta_1 \end{bmatrix} = \begin{bmatrix} 24.3 \\ 143.9 \end{bmatrix}$$

and

$$\begin{bmatrix} 12 & 78 & 650 \\ 78 & 650 & 6084 \\ 650 & 6084 & 60710 \end{bmatrix} \begin{bmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \end{bmatrix} = \begin{bmatrix} 13.586 \\ 91.408 \\ 774.718 \end{bmatrix}$$

The matrix above is solved to get the values of β_i and substitute into (12) and (13) for linear and quadratic models.

Linear model: $f(x) = 0.9916666667 - 0.02161538462x$

Quadratic model: $f(x) = 0.960045455 + 0.035167333x - 0.001042458x^2$

By using the LSCG method, the optimisation problems in (20) and (21) are formed by using MATLAB coding as shown in Figure 3 and Figure 4.

```
syms a b c d p
d = [1 1.032 1.049 1.071 1.105 1.128 1.152 1.195 1.207 1.215 1.201 1.231 1.272];
p = [1 2 3 4 5 6 7 8 9 10 11 12 13];
q=sum(((a+b*p)-d).^2)
all=expand(q)
diff(all,a)
diff(all,b)
```

Figure 3. MATLAB Code of Linear Models

```
syms a b c d p
d = [1 1.032 1.049 1.071 1.105 1.128 1.152 1.195 1.207 1.215 1.201 1.231 1.272];
p = [1 2 3 4 5 6 7 8 9 10 11 12 13];
q=sum(((a+b*p+c*(p.^2))-d).^2)
all=expand(q)
diff(all,a)
diff(all,b)
diff(all,c)
```

Figure 4. MATLAB Code of Quadratic Models

The linear and quadratic models are as follows.

$$f(x) = 12\beta_1^2 + 156\beta_1\beta_2 - \frac{6793\beta_1}{250} + 650\beta_2^2 - \frac{22852\beta_2}{125} + \frac{48287}{3125}$$

$$f(x) = 12\beta_1^2 + 156\beta_1\beta_2 + 1300\beta_1\beta_3 - \frac{6793\beta_1}{250} + 650\beta_2^2 + 12168\beta_2\beta_3 - \frac{22852\beta_2}{125} + 60710\beta_3^2 - \frac{387359\beta_3}{250} + \frac{48287}{3125}$$

These two functions are set as test functions and NTT coefficient with TTCG Method is used to solve these optimisation problems at any random point by using Strong Wolfe line search. The linear and quadratic test function results are then converted into the function form.

Linear Models:

$$f(x) = 0.991303030036635 + 0.021671328701366x$$

Quadratic Models:

$$f(x) = 0.957726821447152 + 0.036061081522954x - 0.001106902551699x^2$$

Linear and quadratic models are also formed through Trendline method. Linear and quadratic graph of the CPI based on year 2010 in Malaysia versus years are generated by using build-in function in Microsoft Excel. The trend line indicates the trend and best fit of the data, and the equations are generated automatically.

From the Figure 5 and Figure 6, the approximate function generated from Trendline method are as below.

Linear Trendline Model: $f(x) = 0.991303030303029 + 0.021671328671329x$

Quadratic Trendline Model: $f(x) = 0.957727272727261 + 0.036060939060942x - 0.001106893106893x^2$

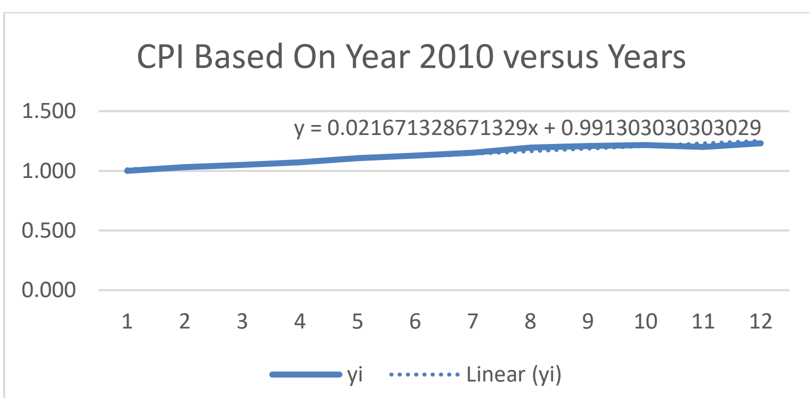

Figure 5. Linear Trend Line for CPI Ratio Based on Year 2010 versus Years in Malaysia

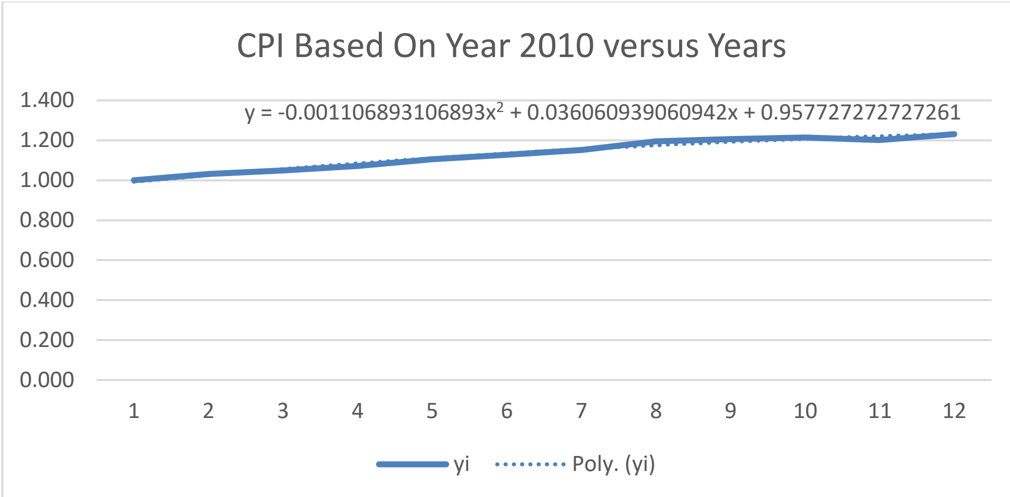

Figure 6. Quadratic Trend Line for CPI Ratio Based on Year 2010 versus Years in Malaysia

By setting data for 2022 as $x = 13$, the estimated inflation rate is calculated using the linear and quadratic models of all three methods above. The relative errors are calculated to compare the most efficient method which could give the least relative error with the actual value. All the approximate functions and relative errors from LG, LSCG and Trendline methods are concluded in Table 3.

Table 3. Estimation Points and Relative Errors of Each Method

Method		**Estimation Point**	**Relative Error**
LS	Linear	1.272666667	0.000524109
	Quadratic	1.241045455	0.024335334
LSCG	Linear	1.273030303	0.000809987
	Quadratic	1.239454350	0.025586203
Trendline	Linear	1.273030303	0.000809987
	Quadratic	1.239454545	0.025586049

Based on the Table 3, the linear function gives the smallest relative error compared to quadratic function. This means that the linear function is more accurate to estimate CPI ratio of Malaysia based on year 2010. Among the linear functions, the LS method gives lesser relative error compared to LSCG and Trendline Method. This may be due to the data used is in small to medium size, the direct calculation method, LS would show numerically stable and precise compared to the iterative method of LSCG which is more complex. However, relative errors are relatively small and similar to each other. The small relative error of linear functions means they can give the closest value to the actual data. Hence, linear LS is the best method to estimate the CPI ratio in Malaysia based on year 2010 but linear LSCG and linear trendline method could also be considered in the process of estimating CPI ratio in Malaysia based on year 2010.

5. CONCLUSION

The TTCG method specifically with the RMIL+, UMW and NTT coefficients are tested with ten test functions selected from [38] under Strong Wolfe line search. NTT CG method shows the least NOI and CPU time and used to estimate the inflation rate in term of CPI ratio of 2022 in Malaysia based on year 2010. Linear NTT CG method with $f(x) = 0.991303030036635 + 0.021671328701366x$ gives the estimation point of 1.273030303 which is same as CPI of 127.3 and inflation rate of 3.3% with the relative error of 0.000809987. As the relative error is small, LSCG method with NTT CG coefficient is efficient to estimate the CPI ratio in Malaysia. However, Linear LS method with the smallest relative error of 0.000524109 is the most suitable method to estimate the inflation rate in Malaysia. This research focuses on comparing the efficiency of RMIL+, UMW and NTT CG Method for solving the unconstrained optimisation problem under Strong Wolfe line search. Further study could focus on exact line search and other inexact line searches such as Armijo rule, Weak Wolfe rule and Goldstein rule. The other new modified CG Methods could also be further studied to study a more accurate method to estimate the inflation rate in Malaysia.

ACKNOWLEDGEMENT

The authors would like to thank the anonymous reviewers for their valuable comments.

FUNDING STATEMENT

The authors received no funding from any party for the research and publication of this article.

AUTHOR CONTRIBUTIONS

Shin Yi Wong: Conceptualization, Data Curation, Methodology, Validation, Writing – Original Draft Preparation;
Nur Syarafina Mohamed: Project Administration, Supervision, Writing – Review & Editing;
Norhaslinda Zullpakkal: Project Administration, Supervision, Writing – Review & Editing.

CONFLICT OF INTERESTS

No conflict of interest was disclosed.

ETHICS STATEMENTS

The data used in this research collected from OpenDOSM, which is an open-source and freely available website responsible for cataloging, visualizing, and analysing data from DOSM. The dataset is named as "Tables Consumer Price Index, 2022" accessed from https://open.dosm.gov.my/publications/cpi_annual_2022. Our publication ethics follow The Committee of Publication Ethics (COPE) guideline. https://publicationethics.org/.

REFERENCES

[1] N. Zullpakkal, N. Aini, N. Ghani, N. Mohamed, N. Idalisa, and M. Rivaie, "Covid-19 data modelling using hybrid conjugate gradient method", *Journal of Information and Optimization Sciences*, vol. 43, no. 4, pp. 837-853, 2022, doi:10.1080/02522667.2022.2060610.

[2] E. K. P. Chong, and S. H. Zak, *An Introduction to Optimization*, 4th ed. Hoboken, NJ, USA: Wiley, 2013.

[3] M. Elhamid, C. Yacine, and T. Bechouat, "Two improved nonlinear conjugate gradient methods with application in conditional model regression function", *Journal of Industrial and Management Optimization*, vol. 21, no. 1, pp. 658-675, 2025, doi:10.3934/jimo.2024098.

[4] I. Griva, S. G. Nash, and A. Sofer, *Linear and Nonlinear Optimization*, 2nd ed. Philadelphia, PA, USA: SIAM, 2008.

[5] Y. Dai, and Y. Yuan, "An efficient hybrid conjugate gradient method for unconstrained optimization", *Annals of Operations Research*, vol. 103, no. 1-4, pp. 33-47, 2001, doi:10.1023/a:1012930416777.

[6] E. Birgin, and J. Martinez, "A spectral conjugate gradient method for unconstrained optimization", *Applied Mathematics & Optimization*, vol. 43, no. 2, pp. 117-128, 2001, doi:10.1007/s00245-001-0003-0.

[7] M. Moller, "A scaled conjugate gradient algorithm for fast supervised learning", *Neural Networks*, vol. 6, no. 4, pp. 525-533, 1993, doi:10.1016/s0893-6080(05)80056-5.

[8] N. Andrei, "A new three-term conjugate gradient algorithm for unconstrained optimization", *Numerical Algorithms*, vol. 68, no. 2, pp. 305-321, 2014, doi:10.1007/s11075-014-9845-9.

[9] N. Shapiee, M. Rivaie, M. Mamat, and P. Ghazali, "A new family of conjugate gradient coefficient with application", *International Journal of Engineering & Technology*, vol. 7, no. 3.28, pp. 36-43, 2018, doi:10.14419/ijet.v7i3.28.20962.

[10] I. M. Sulaiman, and M. Mamat, "A new conjugate gradient method with descent properties and its application to regression analysis," *Journal of Numerical Analysis, Industrial Applications Mathematics*, vol. 12, no. 1-2, pp. 25–39, 2020.

[11] I. Sulaiman, N. Bakar, M. Mamat, B. Hassan, M. Malik, and A. Ahmed, "A new hybrid conjugate gradient algorithm for optimization models and its application to regression analysis", *Indonesian Journal of Electrical Engineering and Computer Science*, vol. 23, no. 2, pp. 1100, 2021, doi:10.11591/ijeecs.v23.i2.pp1100-1109.

[12] N. Pakkal, N. Shapiee, S. Husin, W. Khadijah, N. Salahudin, S. Zokriet et al., "The implementation of conjugate gradient methods for data fitting", *Data Analytics and Applied Mathematics (DAAM)*, pp. 23-29, 2023, doi:10.15282/daam.v4i1.9499.

[13] I. Sulaiman, N. Idalisa, N. Zullpakkal, P. Kaelo, and N. Shapiee, "A descent three-term conjugate gradient direction for problems of unconstrained optimization with application", *International Journal of Computational Methods*, vol. 22, no. 07, 2025, doi:10.1142/s0219876225500100.

[14] H. Khudhur, and R. Mahmood, "Improved conjugate gradient algorithm for removing noise from images", *2025 7th International Congress on Human-Computer Interaction, Optimization and Robotic Applications (ICHORA)*, pp. 1-5, 2025, doi:10.1109/ichora65333.2025.11016997.

[15] A. Ibrahim, B. Fathi, and M. Abdulrazzaq, "Improving three-term conjugate gradient methods for training artificial neural networks in accurate heart disease prediction", *Neural Computing and Applications*, vol. 37, no. 16, pp. 10381-10405, 2025, doi:10.1007/s00521-025-11121-9.

[16] R. Yunus, K. Kamfa, I. Sulaiman, and M. Mamat, "A novel three term conjugate gradient method for unconstrained optimization via shifted variable metric approach with application", *Studies in Systems, Decision and Control*, pp. 581-596, 2022, doi:10.1007/978-3-031-04028-3_37.

[17] A. Wanto, M. Zarlis, S. Sawaluddin, and D. Hartama, "Analysis of artificial neural network backpropagation using conjugate gradient Fletcher Reeves in the predicting process", *Journal of Physics: Conference Series*, vol. 930, pp. 012018, 2017, doi:10.1088/1742-6596/930/1/012018.

[18] Q. C. Shu, "Determinants of foreign direct investment in Malaysia," *Issues and Perspectives in Business and Social Sciences*, no. 2, pp. 115–142, 2024, doi:10.33093/ipbss.2024.4.2.2.

[19] H. Wasi, and M. Shiker, "Proposed CG method to solve unconstrained optimization problems", *Journal of Physics: Conference Series*, vol. 1804, no. 1, pp. 012024, 2021, doi:10.1088/1742-6596/1804/1/012024.

[20] I. Arzuka, M. Bakar, and W. Leong, "A scaled three-term conjugate gradient method for unconstrained optimization", *Journal of Inequalities and Applications*, vol. 2016, no. 1, 2016, doi:10.1186/s13660-016-1239-1.

[21] N. Andrei, "A simple three-term conjugate gradient algorithm for unconstrained optimization", *Journal of Computational and Applied Mathematics*, vol. 241, pp. 19-29, 2013, doi:10.1016/j.cam.2012.10.002.

[22] M. Simionescu, "Machine learning vs. econometric models to forecast inflation rate in Romania? The role of sentiment analysis", *Mathematics*, vol. 13, no. 1, pp. 168, 2025, doi:10.3390/math13010168.

[23] S. K. W. Su and S. V. Soon, "The impact of inflation on economic growth", *Issues and Perspectives in Business and Social Sciences*, vol. 4, no. 2, pp. 187-201, 2024, doi:10.33093/ipbss.2024.4.2.6.

[24] M. Jamil, X. Yang, and H. Zepernick, "Test functions for global optimization", *Swarm Intelligence and Bio-Inspired Computation*, pp. 193-222, 2013, doi:10.1016/b978-0-12-405163-8.00008-9.

[25] K. Hillstrom, "A simulation test approach to the evaluation of nonlinear optimization algorithms", *ACM Transactions on Mathematical Software*, vol. 3, no. 4, pp. 305-315, 1977, doi:10.1145/355759.355760.

[26] N. Gould, and J. Scott, "A note on performance profiles for benchmarking software", *ACM Transactions on Mathematical Software*, vol. 43, no. 2, pp. 1-5, 2016, doi:10.1145/2950048.

[27] E. Dolan, and J. More, "Benchmarking optimization software with performance profiles", *Mathematical Programming*, vol. 91, no. 2, pp. 201-213, 2002, doi:10.1007/s101070100263.

[28] B. Hassan, I. Moghrabi, A. Ibrahim, and H. Jabbar, "Improved conjugate gradient methods for unconstrained minimization problems and training recurrent neural network", *Engineering Reports*, vol. 7, no. 2, 2025, doi:10.1002/eng2.70019.

[29] D. C. Montgomery, E. A. Peck, and G. G. Vining, *Introduction to Linear Regression Analysis*, 6th ed. Hoboken, NJ, USA: Wiley, 2021.

[30] M. Zarandi, M. Zarinbal, A. Zarinbal, I. Turksen, and M. Izadi, "Using type-2 fuzzy function for diagnosing brain tumors based on image processing approach", *International Conference on Fuzzy Systems*, 2010, doi:10.1109/fuzzy.2010.5584469.

[31] J. Wolberg, "Data Analysis Using the Method of Least Squares: Extracting the Most Information from Experiments,", *USA: Springer*, New York, NY, 2006.

[32] Z. Dai, "Comments on a new class of nonlinear conjugate gradient coefficients with global convergence properties", *Applied Mathematics and Computation*, vol. 276, pp. 297-300, 2016, doi:10.1016/j.amc.2015.11.085.

[33] A. Umar, M. Mamat, I. Sulaiman, M. Waziri, and A. Mandara, "A new modification of conjugate gradient parameter with efficient line search for nonconvex function," *International Journal of Scientific & Technology Research*, vol. 9, no. 3, pp. 6572–6575, 2020.

[34] D. Omar, S. Shareef, and B. Fathi, "A new three -term conjugate gradient algorithm for solving minimization problems", *Science Journal of University of Zakho*, vol. 11, no. 4, pp. 544-547, 2023, doi:10.25271/sjuoz.2023.11.4.1152.

[35] W. Y. Wang, D. Gupta, and S. Potthoff, "On evaluating the impact of flexibility enhancing strategies on the performance of nurse schedules", *Health Policy*, vol. 93, no. 2-3, pp. 188-200, 2009, doi:10.1016/j.healthpol.2009.07.003.

[36] R. B. Yunus, A. R. El-Saeed, N. Zainuddin, and H. Daud, "A structured RMIL conjugate gradient-based strategy for nonlinear least squares with applications in image restoration problems", *AIMS Mathematics*, vol. 10, no. 6, pp. 14893-14916, 2025, doi:10.3934/math.2025668.

[37] K. D. Lawrence, R. K. Klimberg, and S. M. Lawrence, "Fundamentals of Forecasting Using Excel,", *USA: Industrial Press*, New York, NY, 2009.

[38] N. Andrei, "An unconstrained optimization test functions collection," *Advanced Modeling and Optimization*, vol. 10, no. 1, pp. 147–161, 2008.

BIOGRAPHIES OF AUTHORS

	**Shin Yi Wong** did her bachelor's degree in Department of Mathematical Science at Universiti Teknologi Malaysia (UTM). She is currently continuing her Master in Mathematics in UTM. Her academic interests focus on operational research. This paper reflects her commitment to exploring innovative ideas and contributing to scholarly discussions within the field. Wong Shin Yi aspires to expanding her knowledge through hands-on experience and continued learning. She can be contacted at email: wong.yi@graduate.utm.my.
	**Nur Syarafina Mohamed** is a Senior Lecturer at the Department of Mathematical Sciences, Universiti Teknologi Malaysia (UTM). She earned her PhD in Mathematics from Universiti Sultan Zainal Abidin (UNISZA) in 2017, following her MSc and BSc in Mathematics from UTM. Her research expertise lies in optimisation and operational research, with applications in mathematical modelling, image processing, and data analysis. With a strong educational foundation and a wealth of published work ranging from algorithm development to real-world applications she actively contributes to both theoretical advancements and practical problem-solving within the mathematical sciences. She can be contacted at email: nursyarafina@utm.my.
	**Norhaslinda binti Zull Pakkal** is a Senior Lecturer in the Faculty of Computer and Mathematical Sciences at UiTM Cawangan Terengganu Kampus Kuala Terengganu. Her research primarily focuses on optimisation, with special expertise in CG methods and their hybrid variants. She has authored several notable papers, including work on the global convergence of new spectral conjugate gradient methods and the application of CG methods for COVID-19 data modelling and image restoration problems. She can be contacted at email: lindazullpakkal@uitm.edu.my.